1.3 Future Value

$FV = P(1+i)^t$

Consider this senario: you invest $1000 over 4 years coumpounded annually at 3%.

After 1 year: $1000(1+0.03)^1 = 1000(1.03)$

After 2 years: $[1000(1.03)](1.03) = 1000(1.03)^2$

After 3 years: $[1000(1.03)^2](1.03) = 1000(1.03)^3$

After 4 years: $1000(1.03)^4 = 1125.51\$$

Hence the equation: $FV = P(1+r)^t$

Now consider the same investment with semi-annually compounding. This means that the half of the interest is paid at 6-month intervals. We identify this interest with the letter *i*.

$\frac{1}{2}$ of 3%

After 6 months (1 compounding period): $FV = 1000(1+0.015)^1$

1 year

2 compounding periods: $= [1000(1.015)](1.015) = 1000(1.015)^2$

3 compounding periods: $= [1000(1.015)^2](1.015) = 1000(1.015)^3$

4 compounding periods: $= 1000(1.015)^4$

... After 48 months (8 compounding periods): $1000(1.015)^8 = 1126.49$

Hence the equation ,

$$FV = P(1+i)^n$$

where *i* is the interest rate per compounding period

and *n* is the total number of compounding periods.

EXAMPLE 2 Determining the future value of an investment with semi-annual compounding

Matt has invested a $23 000 inheritance in an account that earns 13.6%, compounded semi-annually. The interest rate is fixed for 10 years. Matt plans to use the money for a down payment on a house in 5 to 10 years.

always quoted as ANNUAL

a) What is the future value of the investment after 5 years? What is the future value after 10 years?

b) Compare the principal and the future values at 5 years and 10 years. What do you notice?

c) If the investment had earned simple interest, would the relationship between the principal and the future values have been the same? Explain.

$FV = P(1+i)^n$

a) 5 years

$P = 23000$

$i = \frac{13.6\%}{2} = 0.068$

$n = 2 \times 5 = 10$

$FV = 23000(1.068)^{10}$

$FV = 44405.87\$$

10 years

$P = 23000$

$i = .068$

$n = 2 \times 10 = 20$

$FV = 23000(1.068)^{20}$

$FV = 85733.96\$$

b) we have almost twice as much money after 10 yrs compared to 5 years.

c) No... let's compare

5 yrs:

$I = Prt$

$= 23000(0.136)(5)$

$= 15640$

$FV = P + I$

$= 38640\$$

10 yrs:

$I = Prt$

$= 23000(0.136)(10)$

$= 31280$

$FV = 54280\$$

not even close to double

EXAMPLE 4 Comparing interest on investments with different compounding periods

Céline wants to invest $3000 so that she can buy a new car in the next 5 years. Céline has the following investment options:

A. 4.8% compounded annually → $i = 0.048 \rightarrow n = 5$

B. 4.8% compounded semi-annually → $i = 0.024 \rightarrow n = 10$

C. 4.8% compounded monthly → $i = .048 \div 12 = 0.004 \rightarrow 5 \times 12 = 60$

D. 4.8% compounded weekly → $i = .048 \div 52 = 0.000923\ldots \rightarrow 5 \times 52 = 260$

E. 4.8% compounded daily → $i = \frac{0.048}{365} \rightarrow n = 5 \times 365 = 1825$

Compare the interest earned by each of these options for terms of 1 to 5 years.

A. $FV = 3000(1.048)^5 = 3792.52\$$ $+10.43\$$

B. $FV = 3000(1.024)^{10} = 3802.95\$$ $+8.97\$$

C. $FV = 3000(1.004)^{60} = 3811.92\$$ $+1.41\$$

D. $FV = 3000\left(1 + \frac{0.048}{52}\right)^{260} = 3813.33\$$ $+0.36\$$

E. $FV = 3000\left(1 + \frac{0.048}{365}\right)^{1825} = 3813.69\$$

Rule of 72: A good estimate for the doubling time of an investment is found by dividing 72 by the annual interest rate as a percent ... answer is in years; works best with annual compounding

EXAMPLE 5 Estimating doubling times for investments

Both Berta and Kris invested $5000 by purchasing Canada Savings Bonds. Berta's CSB earns 8%, compounded annually, while Kris's CSB earns 9%, compounded annually.

a) Estimate the doubling time for each CSB.
b) Verify your estimates by determining the doubling time for each CSB.
c) Estimate the future value of an investment of $5000 that earns 8%, compounded annually, for 9, 18, and 27 years. How close are your estimates to the actual future values?

a) Berta: 8%
$\frac{72}{8} = 9$ yrs to double her investment

Kris: 9%
$\frac{72}{9} = 8$ yrs to double

b) $FV = 5000(1.08)^9$
$= 9995.02\$$ ← very good estimates

$FV = 5000(1.09)^8$
$= 9962.81$

c) @ 8% it takes about 9 yrs to double an investment
$P = 5000\$$
9 yrs: $FV \approx 10000\$$ } double
18 yrs: $FV \approx 20000\$$ } double
27 yrs: $FV \approx 40000\$$

actual: 9 yrs: $FV = 9995.02\$$
18 yrs: $5000(1.08)^{18} = 19980.10\$$
27 yrs: $5000(1.08)^{27} = 39940.31\$$

Homework: pg. 30 #3, 4, 5, 7, 11, 13