

$$3b) \quad P = 850$$

$$i = 9.2\% = 0.092$$

$$n = 20 \times 12 = 240$$

$$(i) \quad FV = P(1+i)^n$$

$$= 850 \left(1 + \frac{0.092}{12}\right)^{240}$$

$$= 850(6.25\dots)$$

$$= 5314.63 \$$$

$$I = 5314.63 - 850$$

$$I = 4464.63 \$$$

$$i) \quad \frac{72}{9.2} \doteq 7.83 \text{ yrs to double}$$

$$7.83 \text{ yr} = 7 \text{ yr}, 9.9 \text{ months}$$

$$= 7 \text{ yrs}, 10 \text{ months}$$

1.4 Compound Interest Present Value

In 5 years, after graduating from college, Cal wants to spend a year travelling in Canada's three territories. He plans to start in Yukon and then travel east to the Northwest Territories and Nunavut. Cal has determined that he will need at least \$15 000 for his trip. To reach this goal, he wants to invest money now. He has chosen a GIC at 7%, compounded annually.

? How much does Cal need to invest now so that he will have \$15 000 in 5 years?

Strategies: ① guess a starting amount and check the FV → adjust accordingly
 ② rearrange the PV formula

\$10694.80

Present Value: The amount that must be **invested now** to result in a specific **future value** in a certain time at a certain interest rate.

starting with:

set $A = \text{future value}$

$P = \text{Present value}$

$$A = P(1 + i)^n$$

$$\frac{FV}{(1+i)^n} = \frac{PV(1+i)^n}{(1+i)^n}$$

$$PV = \frac{FV}{(1+i)^n}$$

EXAMPLE 2

Determining the present value of an investment that is compounded quarterly

Agnes and Bill are musicians. They have researched the costs to set up a small recording studio. They estimate that \$40 000 will pay for the soundproofing, recording equipment, and computer hardware and software that they need. They plan to set up the studio in 3 years and have invested money at 9.6%, compounded quarterly, to save for it.



- How much money should they have invested?
- How much interest will they earn over the term of their investment?

$$\begin{aligned} a) \quad & FV = 40\,000 \\ & n = 3 \times 4 = 12 \\ & i = \frac{0.096}{4} = 0.024 \end{aligned} \quad \left. \vphantom{\begin{aligned} FV = 40\,000 \\ n = 3 \times 4 = 12 \\ i = \frac{0.096}{4} = 0.024 \end{aligned}} \right\} \quad PV = \frac{FV}{(1+i)^n}$$

$$= \frac{40\,000}{(1.024)^{12}}$$

$$= 30\,092.65538\dots \$$$

They need to invest 30 092.66\$

$$\begin{aligned} b) \quad I &= FV - PV \\ &= 40\,000 - 30\,092.66 \\ &= 9\,907.34\$ \end{aligned} \quad \longrightarrow \quad \text{They will earn } 9\,907.34\$ \text{ in interest}$$

EXAMPLE 3 | Determining an unknown interest rate and unknown term

Laura has invested \$15 500 in a Registered Education Savings Plan (RESP). She wants her investment to grow to at least \$50 000 by the time her newborn enters university, in 18 years.



- a) What interest rate, compounded annually, will result in a future value of \$50 000? Round your answer to two decimal places.
- b) Suppose that Laura wants her \$15 500 to grow to at least \$60 000 at the interest rate from part a). How long will this take?

$$\begin{array}{l}
 a) \quad FV = 50\,000 \\
 \quad \quad PV = 15\,500 \\
 \quad \quad n = 18 \\
 \quad \quad i = ?
 \end{array}
 \left.
 \begin{array}{l}
 FV = PV(1+i)^n \\
 \frac{50\,000}{15\,500} = \frac{15\,500}{15\,500}(1+i)^{18} \\
 \sqrt[18]{3.2258\dots} = \sqrt[18]{(1+i)^{18}}
 \end{array}
 \right\}$$

S
A
M
P
L
E

$$1.067229\dots = 1 + i$$

$$0.067229\dots = i$$

∴ She needs an interest rate of 6.72%

$$\begin{array}{l}
 b) \quad FV = 60\,000 \\
 \quad \quad PV = 15\,500 \\
 \quad \quad i = 0.0672 \\
 \quad \quad n = ?
 \end{array}
 \left.
 \begin{array}{l}
 60\,000 = 15\,500(1+0.0672)^n \\
 \frac{60\,000}{15\,500} = \frac{15\,500}{15\,500}(1.0672)^n \\
 3.87096\dots = 1.0672^n
 \end{array}
 \right\}$$

$$3.87096\dots = 1.0672^n$$

hmm... how do we solve for an exponent?

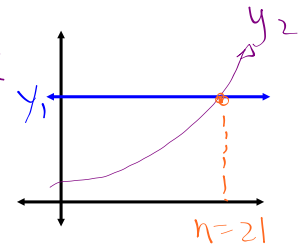
option 1: guess and check
we know $n=18$ gets us \$50 000 so...

try $n=20$: $1.0672^{20} = 3.672\dots$
not enough

try $n=21$: $1.0672^{21} = 3.9188\dots$
so we're over but we have our FV of at least 60 000 when $n=21$ years

option 2:
graph

$$\begin{array}{l}
 y_1 = 3.87096 \\
 y_2 = 1.0672^x
 \end{array}$$



Homework: pg. 40 #5, 6, 7, 9, 10, 14 + quiz 1.1-1.3 tomorrow