

ex// 6% compounded quarterly
for 5 yrs, $P = 100$ \$

$$FV = P(1+i)^n$$

$$n = 5 \text{ yrs} \times 4 \text{ periods/yr} = 20$$

$$i = \frac{6\%}{4} = \frac{.06}{4} = 0.015$$

$$FV = 100(1+0.015)^{20}$$

#6. $P = 1000$ \$ A) $P = 1050$ \$

$$i = 0.05$$

$$n = 5$$

$$FV = 1000(1.05)^5$$

$$= 1276.28$$

$$FV = 1050(1.05)^5$$

$$= \underline{1340.10}$$

B) $i \rightarrow 0.06$

$$= \underline{1338.22}$$

C) $n \rightarrow 5 \times 12$

$$i \rightarrow \frac{0.05}{12}$$

$$= \underline{1283.36}$$

D) $n \rightarrow 6$

$$= \underline{1340.10}$$

#6. $FV = 17500$

$$t = 10$$

$$n = 10 \times 2$$

$$i = \frac{5.6\%}{2} = \frac{.056}{2} = .028$$

$$FV = P(1+i)^n$$

$$PV = \frac{FV}{(1+i)^n}$$

$$= \frac{17500}{(1.028)^{20}}$$

$$= 10673.39$$

1.5 Investments involving regular payments

When a regular payment is made to an investment, we call that an **annuity**. Since this can be complicated to calculate, we'll use the TVM-solver on your calculators. It is important to know how it works.

$$\begin{array}{ll}
 N = \# \text{ of compound periods} & I\% = \text{annual interest rate in \%} \\
 * PV = \text{present value} & * PMT = \text{regular payment} \\
 \quad * \text{money leaving your hands must be } (-) & \\
 * FV = \text{future value} & P/Y = \text{payments per year} \\
 \\
 C/Y = \# \text{ of compound periods per year} & PMT: \text{end} \text{ begin unless otherwise stated, investments happen at the end of a period}
 \end{array}$$

EXAMPLE 1

Determining the future value of an investment involving regular deposits

Darva is saving for a trip to Australia in 5 years. She plans to work on a student visa while she is there, so she needs only enough money for a return flight and her expenses until she finds a job. She deposits \$500 into her savings account at the end of each 6-month period from what she earns as a server. The account earns 3.8%, compounded semi-annually. How much money will be in the account at the end of 5 years? How much of this money will be earned interest?



$$N = 5 \times 2 = 10$$

$$I\% = 3.8$$

$$PV = 0 \rightarrow \text{there is no Principal investment}$$

$$PMT = -500 \quad \$$$

$$FV = \text{Alpha Solve} = 5449.90$$

$$P/Y = 2$$

$$C/Y = 2$$

END

How much is interest?
 \rightarrow she invested $10 \times 500\$ = 5000$

$$\therefore 5449.90 - 5000$$

$$I = 449.90$$

EXAMPLE 2

Comparing a regular payment investment with a single payment investment

Adam made a \$200 payment at the end of each year into an investment that earned 5%, compounded annually. Blake made a single investment at 5%, compounded annually. At the end of 5 years, their future values were equal.

- a) What was their future value?
- b) What principal amount did Blake invest 5 years ago?
- c) Who earned more interest? Why?

a) ADAM

$$N = 5$$

$$I = 5\%$$

$$PV = 0$$

$$PMT = -200$$

$$FV = 1105.13$$

$$P/Y = 1$$

$$C/Y = 1$$

$$END$$

$$F = 1105.13 - 1000$$

$$= 105.13 \$$$

b) BLAKE

$$N = 5$$

$$I = 5$$

$$PV = \text{Solve} = -865.70$$

$$PMT = 0$$

$$FV = 1105.13$$

$$P/Y = 1$$

$$C/Y = 1$$

$$END$$

} must be equal if
no PMTs are made

Blake earns more since his investment matured for longer ... he also invested all at the beginning

c) ADAM : $I = 105.13 \$$

Blake : $1105.13 - 865.70$
 $= 239.23 \$$

EXAMPLE 3

Determining the interest rate of a regular payment investment

Jeremiah deposits \$750 into an investment account at the end of every 3 months. Interest is compounded quarterly, the term is 3 years, and the future value is \$10 059.07. What annual rate of interest does Jeremiah's investment earn?

$$N = 4 \times 3 = 12$$

$$I = 8.00\%$$

$$PV = 0$$

$$PMT = -750$$

$$FV = 10059.07$$

$$P/Y = 4$$

$$C/Y = 4$$

$$END$$

EXAMPLE 4**Determining the regular payment amount of an investment**

Celia wants to have \$300 000 in 20 years so that she can retire. Celia has found a trust account that earns a fixed rate of 10.8%, compounded annually.

- a) What regular payments must Celia make at the end of each year to meet her goal of \$300 000?
 b) How much interest will she earn over the 20 years?

$$a) N = 20$$

$$I = 10.8$$

$$PV = 0$$

$$PMT = -4781.10$$

$$FV = 300\,000$$

$$P/Y = 1$$

$$C/Y = 1$$

END

$$b) \text{ she pays } = (4781.10)(20) \\ = 95\,622 \text{ \$}$$

$$I = 300\,000 - 95\,622 \\ = \$204\,378$$

EXAMPLE 5**Determining the term of a regular payment investment**

On Luis's 20th birthday, he started making regular \$1000 payments into an investment account at the end of every 6 months. He wants to save for a down payment on a home. His investment earns 3.5%, compounded semi-annually.

At what age will he have more than \$18 000?

$$N = 15.78 \dots$$

$$I = 3.5$$

$$PV = 0$$

$$PMT = -1000$$

$$FV = 18\,000$$

$$P/Y = 2$$

$$C/Y = 2$$

END

Since Luis needs 16 periods to grow his investment, he actually need 8 yrs (semi-annually)
 \therefore Luis will be 28.