

Piecewise Functions

Definition An overall function defined by different equations for different intervals of x .

Beware of open/closed points at boundaries.

Example 1:

Muffins the Penguin has started a business where he rents out graphing calculators. He charges a flat rate of 5€ for the first hour, then 7€ for above 1h up to 3 hours and then a flat rate of 8€ plus 2€ for each hour after 3 hours

Graph this function, and describe it using an equation.

$$\text{@ } t=0, \text{ €} = 0$$

$$\text{1st hour, } C = 5 \text{ €}$$

$$\text{1-3 hrs, } C = 7 \text{ €}$$

$$\text{after 3 hrs, } C = 8 + 2t$$

* $m = 2$ and passes through $(3, 8)$

$$y = mx + b$$

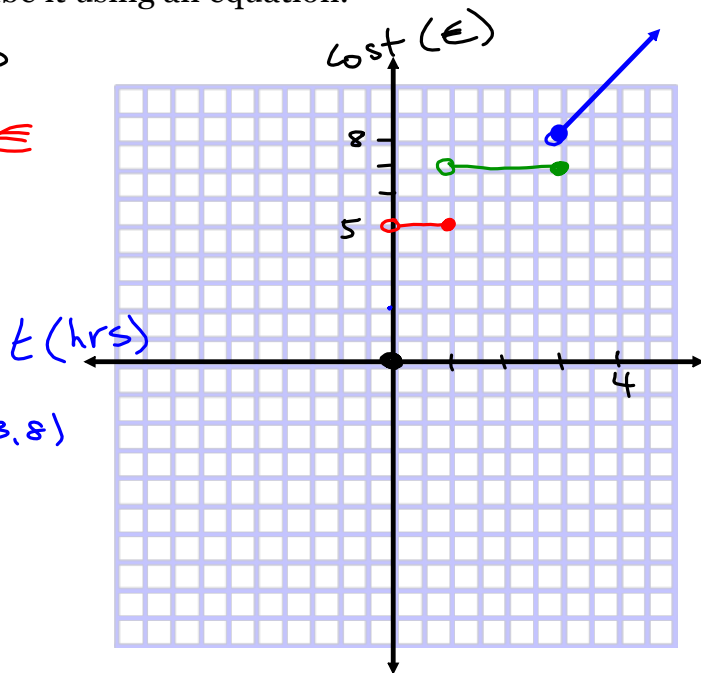
$$8 = 2(3) + b$$

$$8 - 6 = b$$

$$b = 2$$

$$\boxed{y = 2x + 2}$$

$$c(t) = \begin{cases} 0, & t = 0 \\ 5, & 0 < t \leq 1 \\ 7, & 1 < t \leq 3 \\ 2x + 2, & t > 3 \end{cases}$$



Graphing a Piecewise Function

To graph, consider each part of the function on its interval of x ...
 may require multiple TOVs

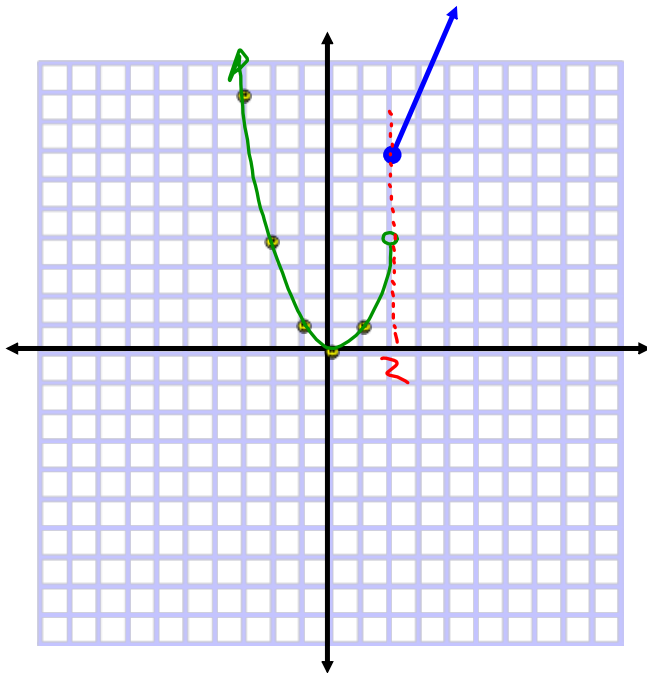
Example 2

$$f(x) = \begin{cases} x^2, & x < 2 \\ 2x+3, & x \geq 2 \end{cases}$$

x	y
2	4

x	y
2	7
3	9
4	11

not continuous
 @ $x = 2$



Testing for continuity

check to make sure the function has the same y -values on the left & right of a boundary

If identical \rightarrow continuous

Example 3: Test if the function $g(x)$ is continuous

$$g(x) = \begin{cases} x+1, & x \leq 0 \\ 2x+1, & 0 < x < 3 \\ 4-x^2, & x \geq 3 \end{cases}$$

left (right boundary)

$$y = x + 1$$

$$y = (0) + 1$$

$$y = 1$$

continuous here!

middle (left): $x = 0$

$$y = 2(0) + 1$$

$$y = 1$$

middle (right): $x = 3$

$$y = 2(3) + 1$$

$$y = 7$$

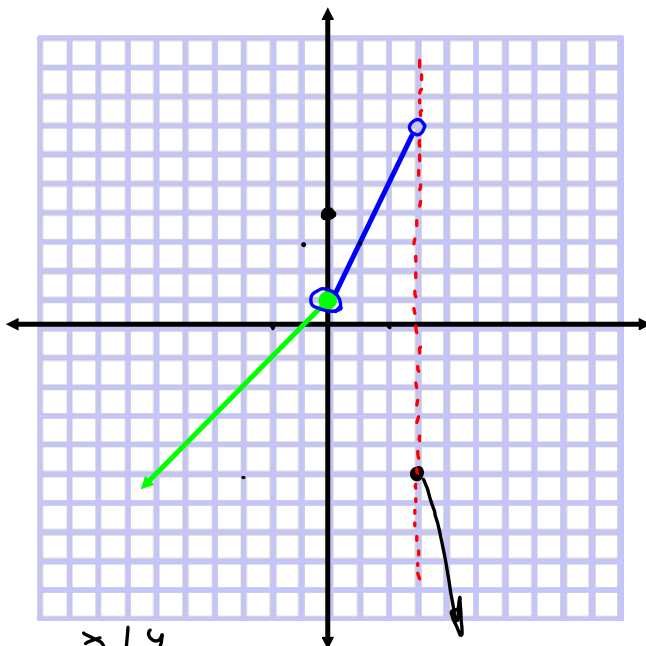
right (left): $x = 3$

$$y = 4 - (3)^2$$

$$= 4 - 9$$

$$y = -5$$

discontinuous
Booo!



$y = x^2$

x	y
0	0
1	1
2	4
3	9
4	16

Red arrows point from the y-values to the right, with labels: +1, +3, +4, +7.

