11.1 Permutations (part 2)

If we have a permutation but several of the items are not distinct (ie. they are the same), we must consider those repetitions. Remember the word Canada?

> if we are arranging *n* elements and there are *a* identical elements, *b* different identical elements and c identical elements of a third type then there are

$\frac{n!}{a!b!c!}$ different ways of arranging the elements

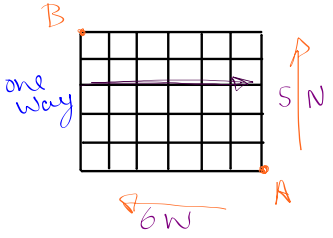
ex. How many distinct arrangements can be formed with the letters of the word Grottoli?

$\frac{8!}{2!2!} = 10\,080$ (2!: o's, 2!: t's)

ex. How about the word Mississippi?

$\frac{11!}{4!4!2!} = 34\,650$ (4!: s's, 4!: i's, 2!: p's)

ex. Hannah is leaving Baked and wants to end up at the corner of 6th and Black street. To get there she must walk 6 blocks to the West and 5 blocks North. We'll assume this makes a perfect grid. In how many different ways can she get there? She can only go West and North.



Think about this as a word

NNNNNWWWWWW

$\frac{11!}{5!6!} = 462$

with one-way $\frac{10!}{5!5!} = 252$

ex. Five people (A, B, C, D, E) are sitting on a bench. In how many ways can we arrange them if:

a) E sits in the middle? 4 3 E 2 1 $= 4! = 24$

b) A and B sit next to each other?

Consider AB as one element } with 2! arrangements

4 3 2 AB (2!) $= 4!\,2! = 48$

c) A and B do NOT sit next to each other?

we could create all possible seating scenarios and add them up ... OR since the # of ways where they DO sit together we know ... subtract that number (48) from all possibilities

$5! - 48 = 72$

$5!$: all possible arrangements; 48: ways they are together

Your Turn

How many ways can one French poster, two mathematics posters, and three science posters be arranged in a row on a wall if

a) the two mathematics posters must be together on an end?

b) the three science posters must be together?

c) the three science posters cannot all be together?

a) MM _ _ _ _ or here
$4!2!2! = 96$
4 spots, 2 ends for MM, 2 arrangements for MM

b) SSS _ _ _
$4!3! = 144$
4 spots, 3! ways for 3 posters

c) All – together
$6! - 144$
$= 576$

Problems involving cases

ex. How many even numbers greater than 300 can you make with three of the digits from the group: 1, 2, 3, 4, 5 and 6 if no number can be repeated?

Consider 2 cases
start with one of these

case 1: # starts with 3 or 5

$2 \times 4 \times 3 = 24$
3 or 5; other #s; 3 even #s to choose from

case 2: start with 4 or 6

$2 \times 4 \times 2 = 16$
4 or 6; other #s; 2 even #s to choose from

∴ total ways $= 24 + 16 = 40$

Your Turn

How many 4-digit odd numbers can you make using the digits 1 to 7 if the numbers must be less than 6000? No digits are repeated.

1 2 3 4 5 6 7

must start with one of these; must end with one of these

case 1: start with 1, 3 or 5

$3 \times 5 \times 4 \times 3 = 180$
1, 3 or 5; leftovers; 3 odds left

case 2: start with 2 or 4

$2 \times 5 \times 4 \times 4 = 160$
2 or 4; leftovers; 4 odds to choose from

total $= 180 + 160 = 340$ ways

Homefun: pg. 524 # 8b, 9, 11-17, 20, 22ac, 24, 25, C3, C5