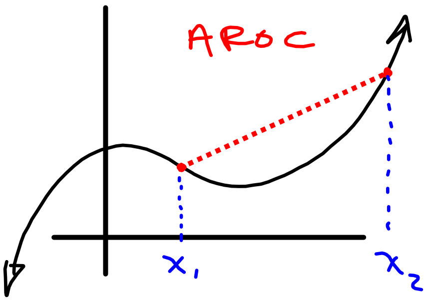
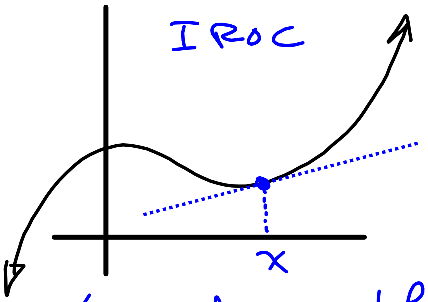
2.2 INSTANTANEOUS RATE OF CHANGE (IROC)

AROC - Average rate of change = slope from $x_1 \rightarrow x_2$

IROC - Instantaneous rate of change at some value of x... we are trying to approximate, as best we can, the slope of the TANGENT @ x.



AROC $[x_1, x_2]$



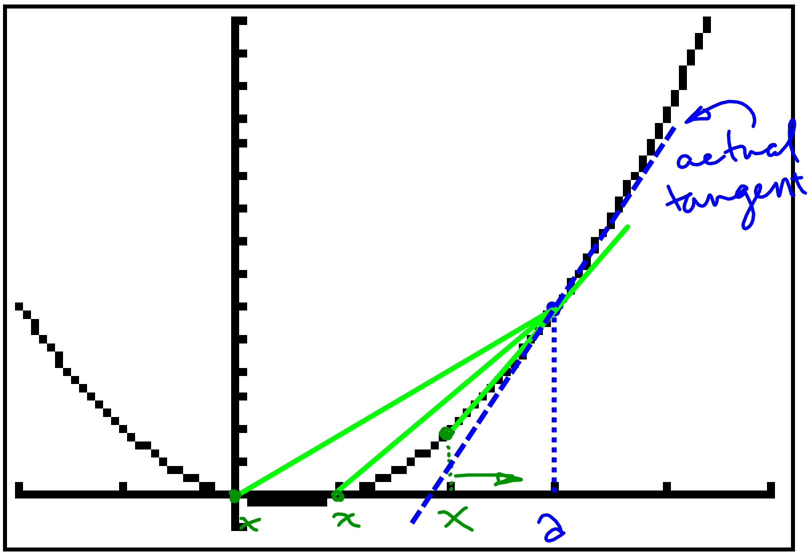
(speed on the speedometer)

* instantaneous slope.

THE METHOD (to the madness??)

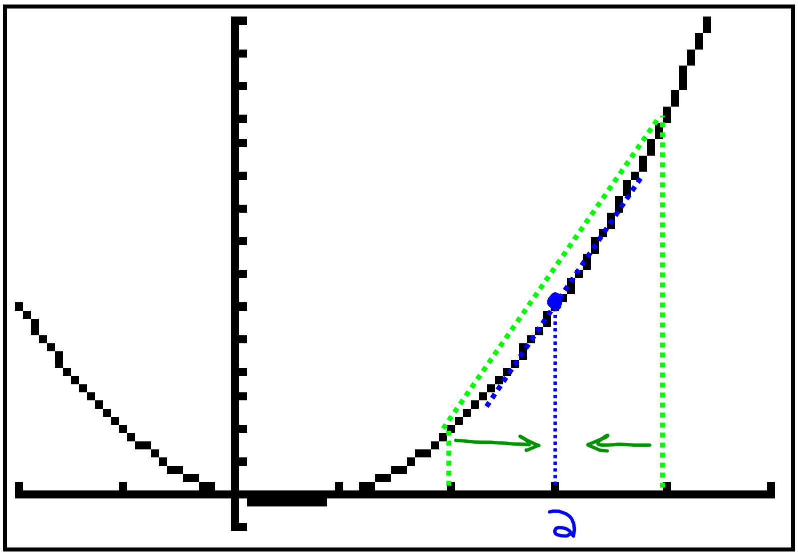
Method 1 - Preceding and Following Intervals

* for AROCs before & after the desired x-value, use intervals approaching zero to estimate IROC



Method 2 - Centred Intervals

Find AROCs for intervals centered about x=a and then make the intervals smaller.



Using Table of Values

Example 1 - Find the IROC of distance (the speed) at 6.4 seconds

Time, t (s)	6.0	6.2	6.4	6.6	6.8	7.0
Distance, $d(t)$ (cm)	208.39	221.76	235.41	249.31	263.46	277.84

Method 1 : before & after

$AROC_{6 \to 6.4} = \frac{y_2 - y_1}{x_2 - x}$

$= \frac{235.41 - 208.39}{6.4 - 6.0}$

$= 67.55$ cm/s

$AROC_{6.2 \to 6.4} = 68.25$ cm/s

$AROC_{6.4 \to 6.8} = 70.125$ cm/s

$AROC_{6.4 \to 6.6} = 69.5$ cm/s

$IROC = \frac{68.25 + 69.5}{2}$

$= 68.875$ cm/s

Method 2 : centred

$AROC_{6 \to 6.8} = 68.8375$ cm/s

$AROC_{6.2 \to 6.6} = 68.875$ cm/s

$\boxed{IROC_{6.4} = 68.875 \text{ cm/s}}$

Example 2 - Find the IROC of temperature at 5 minutes

Time (min)	0	1	2	3	4	5	6	7	8	9	10
Temperature (ºF)	70	125	170	210	250	280	310	335	360	380	400

$AROC_{2-8} = \frac{360 - 170}{8 - 2} = 31.66$ °F/min

$AROC_{3 \to 7} = \frac{335 - 210}{7 - 3} = 31.25$ °F/min

$AROC_{4 \to 6} = \frac{310 - 250}{6 - 4} = 30$ °F/min

$\therefore IROC_5 = 30$ °F/min

Using Equations

Example 1

The population of a small town appears to be growing exponentially. Town planners think that the equation $P(t) = 35\,000\,(1.05)^t$, where $P(t)$ is the number of people in the town and t is the number of years after 2000, models the size of the population. Estimate the instantaneous rate of change in the population in 2015.

Method 1 could skip this

$AROC_{14\to15} = \frac{P(15)-P(14)}{15-14}$

$= 3464.88$ pers/yr.

$AROC_{14.5\to15} = 3507.14$

$AROC_{15\to15.5} = 3593.75$

$\therefore IROC_{15} = \frac{3507.14+3593.75}{2}$

$= 3550$ ppl/yr.

Method 2

$AROC_{14.9\to15.1}$

$= \frac{P(15.1)-P(14.9)}{15.1-14.9}$

$= \frac{\left[35000(1.05)^{15.1} - 35000(1.05)^{14.9}\right]}{0.2}$

$= 3550.1$ ppl/yr

$\boxed{\therefore IROC_{15} = 3550 \text{ ppl/yr}}$

Example Deux

The volume of a cubic crystal, grown in a laboratory, can be modelled by $V(x) = x^3$, where $V(x)$ is the volume measured in cubic centimetres and x is the side length in centimetres. Estimate the instantaneous rate of change in the crystal's volume with respect to its side length when the side length is 5 cm.

method 2:

$AROC_{4.9\to5.1}$

$= \frac{(5.1)^3-(4.9)^3}{5.1-4.9}$

$= 75.01$ cm^3/cm

$AROC_{4.99\to5.01}$

$= \frac{(5.01)^3-(4.99)^3}{5.01-4.99}$

$= 75.001$ cm^3/cm

$\therefore \boxed{IROC_5 = 75 \text{ cm}^3/\text{cm}}$

* w/ calculus take derivative $V(x) = x^3$

$V'(x) = 3x^2$

$V'(5) = 3(5)^2$

$= 75$

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