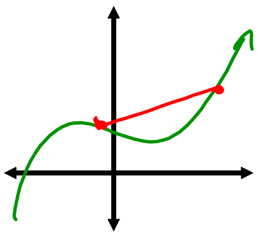
2.3 The Difference Quotient

AROC - slope of a secant (slope b/w 2 points)

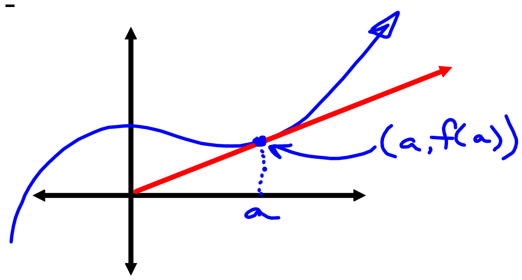
IROC - slope of a tangent (slope **AT** a point)

Graphically...

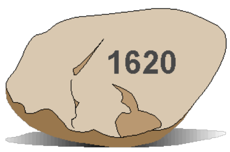
AROC -



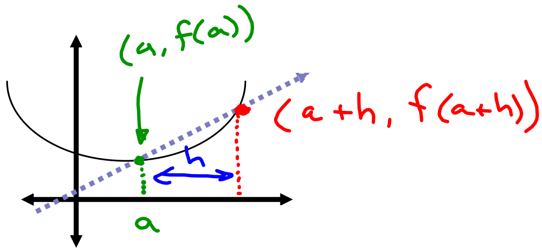
IROC -



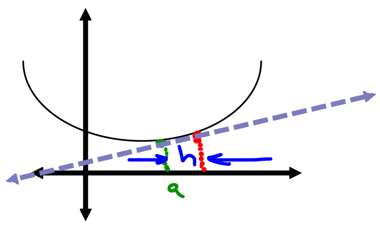
Pictorally



Secant to Tangent

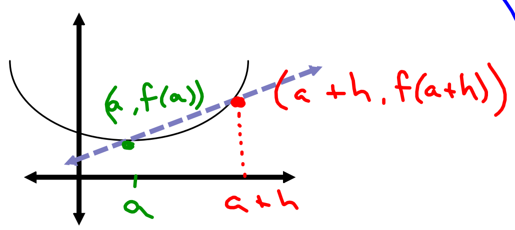


we can imagine the secant becoming a tangent as h diminishes to zero



∴ If we can express the slope of the secant in terms of f(x), a and h, we can then approximate the slope of the tangent (IROC) by using very tiny values of h.

The Difference Quotient



$$m_{secant} = \frac{f(a+h) - f(a)}{(a+h) - a}$$

$$\boxed{DQ = \frac{f(a+h) - f(a)}{h}}$$

For small values of h, use the DQ for the IROC of f(x) @ x=a

$$\boxed{h = 0.0001}$$

IROC aka DQ

Example 1:

Find the slope of the tangent to $f(x) = 6x^2 + 8x$ at $x = 2$

$a = 2$
$h = 0.001$

$f(2.001) = 40.032006$
$f(2) = 40$

$$DQ = \frac{f(a+h) - f(a)}{h}$$
$$= \frac{f(2.001) - f(2)}{0.001}$$
$$= \frac{40.032006 - 40}{0.001}$$
$$= 32.006$$

$\therefore IROC_2 = 32$

Example 2:

Estimate the equation of the tangent line to $y = x^2 - x$ at $x = 3$

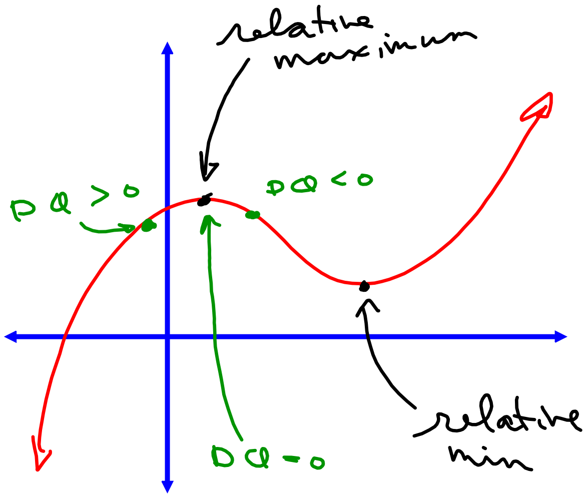
$a = 3$
$h = 0.0001$
$f(3) = 6$
$f(3.0001) = 6.00050001$

$$DQ = \frac{f(3.0001) - f(3)}{0.0001}$$
$$= 5.0001$$

$\therefore m = 5.0001$
pt $(3, 6)$
$y = mx + b$
$6 = 5.0001(3) + b$
$b = -9.003$

$\therefore$ the tangent @ $x = 3$ is approximately
$y = 5.0001x - 9.003$

A look ahead... THE DQ at a minimum or maximum



∴ when the DQ changes from +ve to −ve, we have a relative max value

DQ goes from −ve to +ve → max value

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