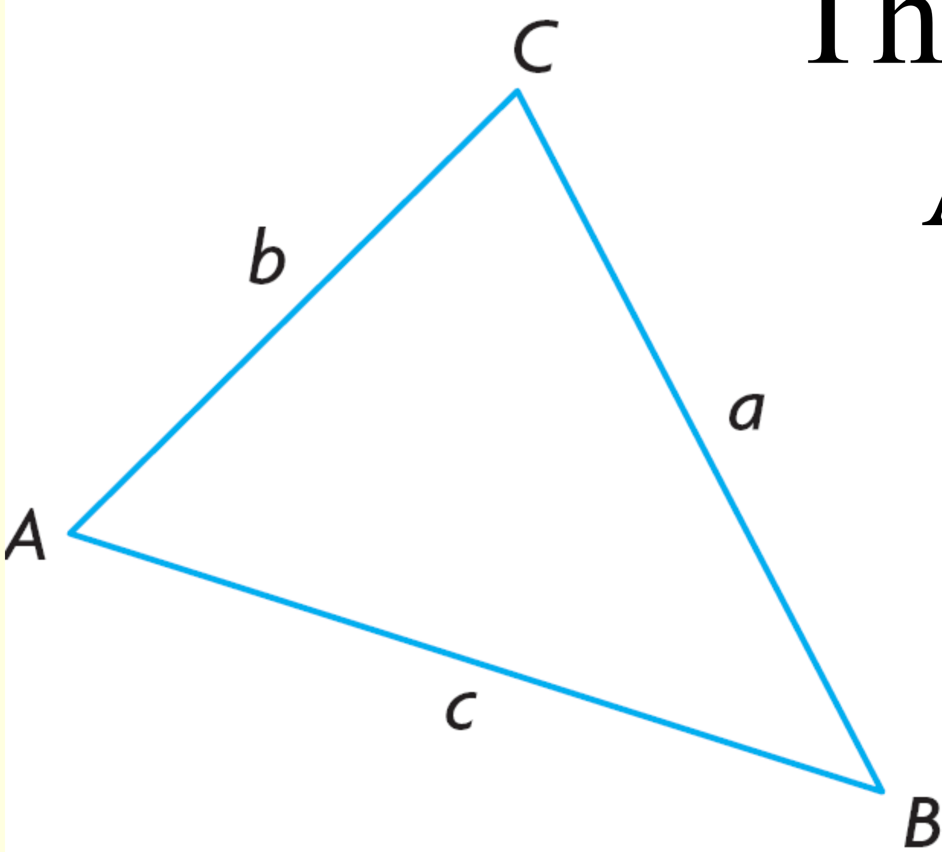
The Sine Law and the Ambiguous Case



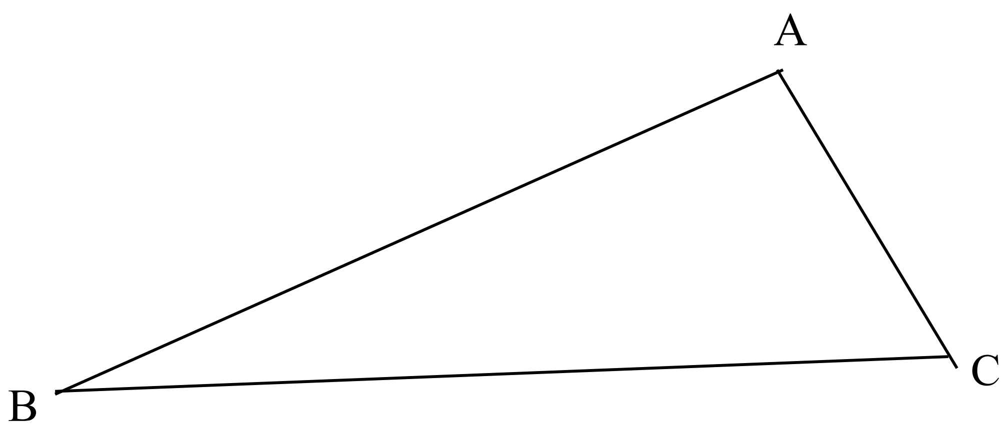
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Or:

For example:

Solve the triangle with:

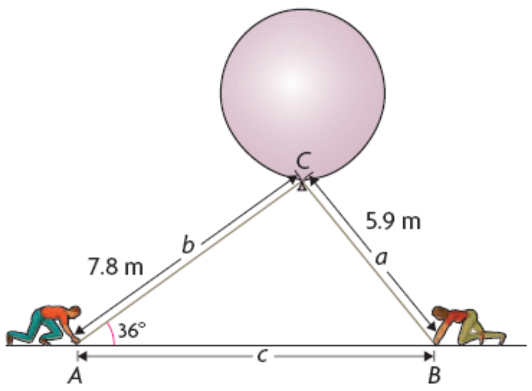
$$b = 3.0 \text{ cm},\ c = 5.5 \text{ cm},\ \angle B = 30^\circ$$

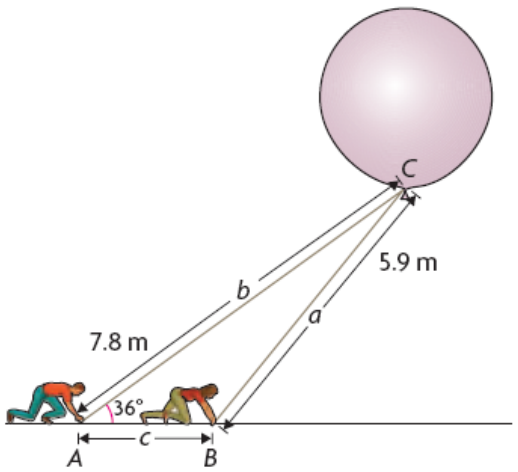


2.3 The Ambiguous Case

Albert and Belle are part of a scientific team studying thunderclouds. The team is about to launch a weather balloon into an active part of a cloud. Albert's rope is 7.8 m long and makes an angle of 36° with the ground. Belle's rope is 5.9 m long.

How far, to the nearest tenth of a metre, is Albert from Belle? (Draw a sketch first.)





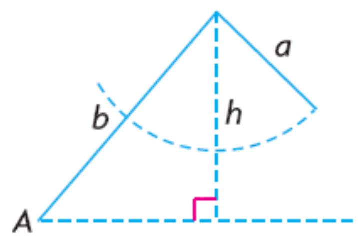
Ah-ha! We have two possibilities - This is called the "Ambiguous Case of the Sine Law"

- The ambiguous case arises in a SSA (side, side, angle) triangle. In this situation, depending on the size of the given angle and the lengths of the given sides, the sine law calculation may lead to 0, 1, or 2 solutions.

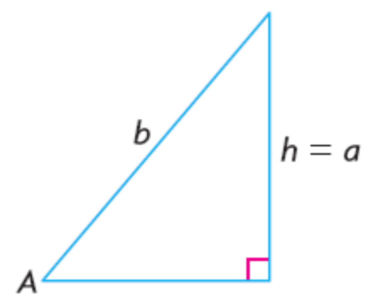
Need to Know

- In the ambiguous case, if $\angle A$, a, and b are given and $\angle A$ is acute, there are four cases to consider. In each case, the height of the triangle is $h = b \sin A$.

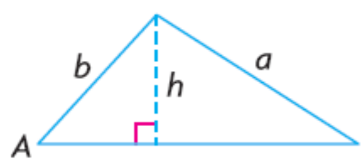
If $\angle A$ is acute and $a < h$, no triangle exists.



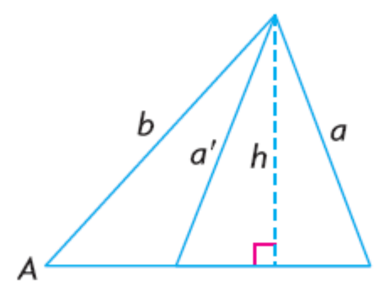
If $\angle A$ is acute and $a = h$, one right triangle exists.



If $\angle A$ is acute and $a > b$, one triangle exists.



If $\angle A$ is acute and $h < a < b$, two triangles exist.



For example, given $\triangle ABC$, where $\angle A = 36°$, $a = 7.0$ cm, and $c = 10.4$ cm, there possible triangles:

The Pont du Gard near Nîmes, France, is a Roman aqueduct. An observer in a hot-air balloon some distance away from the aqueduct determines that the angle of depression to each end is $54°$ and $71°$, respectively. The closest end of the aqueduct is 270.0 m from the balloon. Calculate the length of the aqueduct to the nearest tenth of a metre.

Pg 108 #5, 6, 8, 9, 17, 24