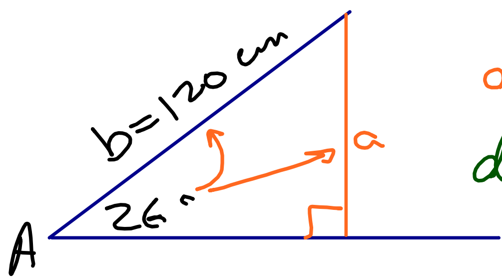
SOH



b) $\sin 26° = \frac{a}{120}$

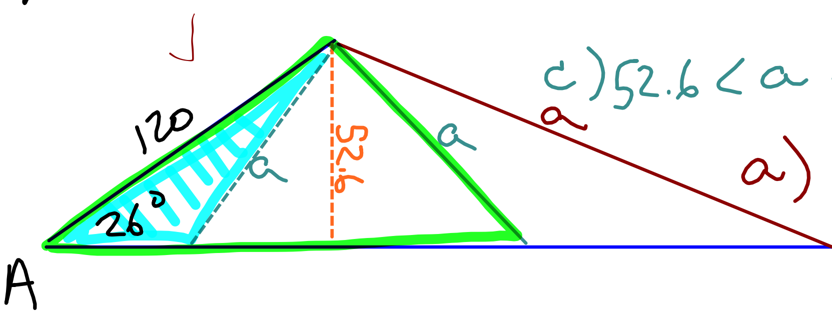
$a = 52.6$ cm

$\angle A = 26°$

$b = 120$ cm

$a = ?$

d) no triangle if $a < 52.6$

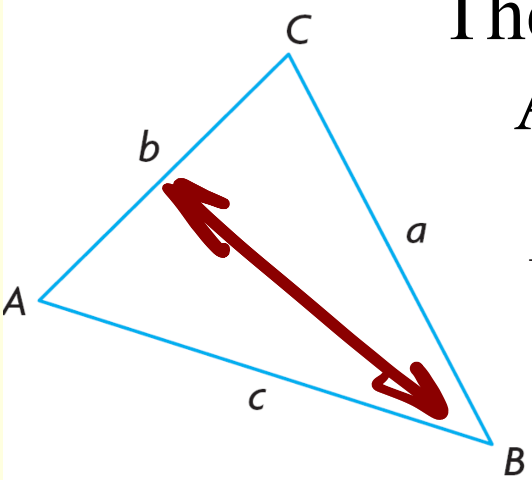


c) $52.6 < a < 120$

a) $a > 120$

↳ one oblique △

The Sine Law and the Ambiguous Case



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

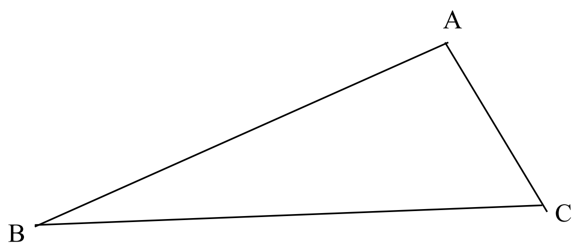
Or:

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

For example:

Solve the triangle with:

$$b = 3.0 \text{ cm},\ c = 5.5 \text{ cm},\ \angle B = 30^\circ$$

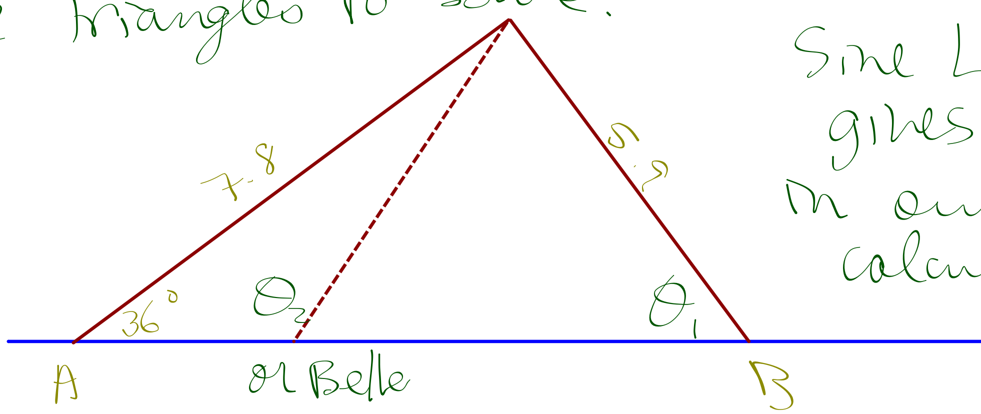


2.3 The Ambiguous Case

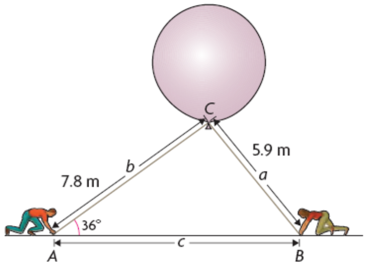
Albert and Belle are part of a scientific team studying thunderclouds. The team is about to launch a weather balloon into an active part of a cloud. Albert's rope is 7.8 m long and makes an angle of 36° with the ground. Belle's rope is 5.9 m long.

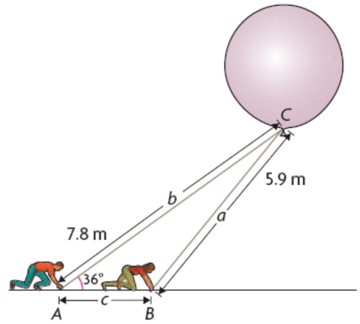
How far, to the nearest tenth of a metre, is Albert from Belle? (Draw a sketch first.)

Since the information is presented as Angle, Side, Side (ASS), you may have 2 triangles to solve.



Sine Law gives θ_1 in our calculators

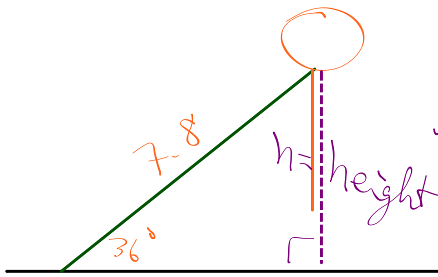




Ah-ha! We have two possibilities - This is called the "Ambiguous Case of the Sine Law"

- The ambiguous case arises in a SSA (side, side, angle) triangle. In this situation, depending on the size of the given angle and the lengths of the given sides, the sine law calculation may lead to 0, 1, or 2 solutions.

There can be zero solutions?



what length of rope does Belle need to complete the Δ

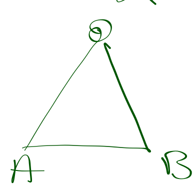
calculate the height and compare to the given length of 5.9 (in this case)

$\sin 36° = \frac{h}{7.8}$

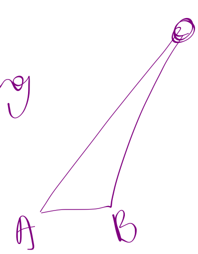
$h = 7.8 \sin 36°$

$\boxed{h = 4.58 \text{ m}}$

since Belle's rope is longer than 4.58 m, she could be standing 10 m away



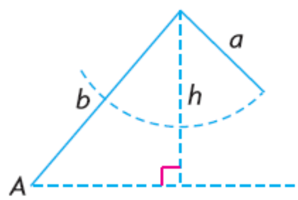
But, also since Belle's rope is shorter than Albert's, she can be standing 2.6 m away



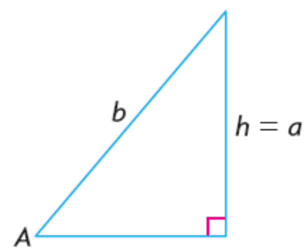
Need to Know

- In the ambiguous case, if $\angle A$, a, and b are given and $\angle A$ is acute, there are four cases to consider. In each case, the height of the triangle is $h = b \sin A$.

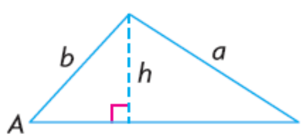
If $\angle A$ is acute and $a < h$, no triangle exists.



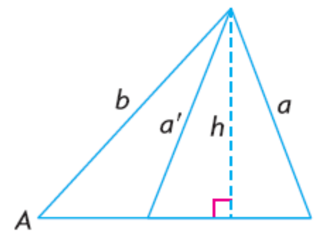
If $\angle A$ is acute and $a = h$, one right triangle exists.



If $\angle A$ is acute and $a > b$, one triangle exists.

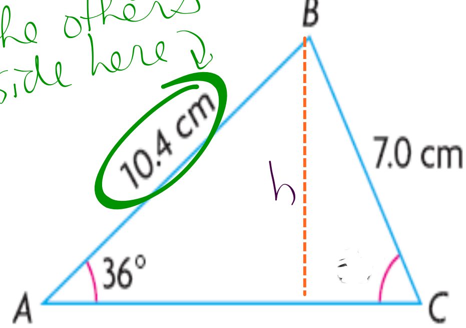


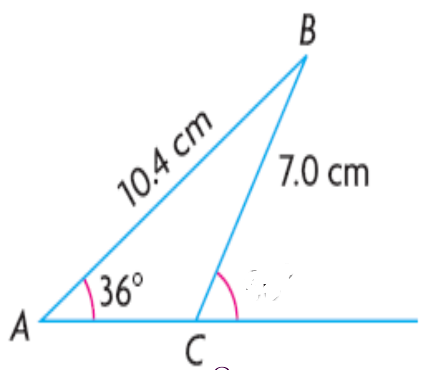
If $\angle A$ is acute and $h < a < b$, two triangles exist.



For example, given ΔABC, where $\angle A = 36°$, $a = 7.0$ cm, and $c = 10.4$ cm, there   possible triangles:

always place the others side here





How do we know there are 2 cases?

$\sin 36° = \frac{h}{10.4}$

$h = 10.4 \sin 36°$

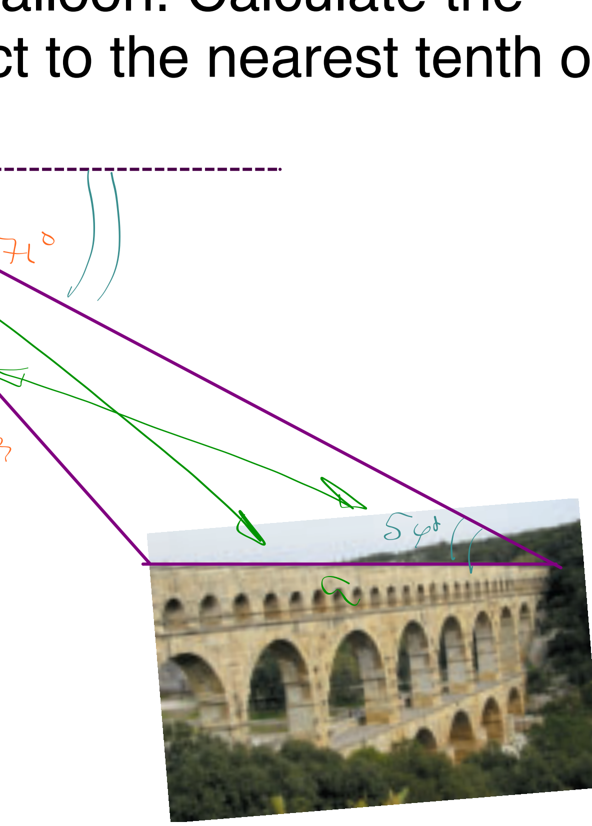
$h \doteq 6.1$ cm

since $6.1 < 7.0 < 10.4$

$h < a < c$

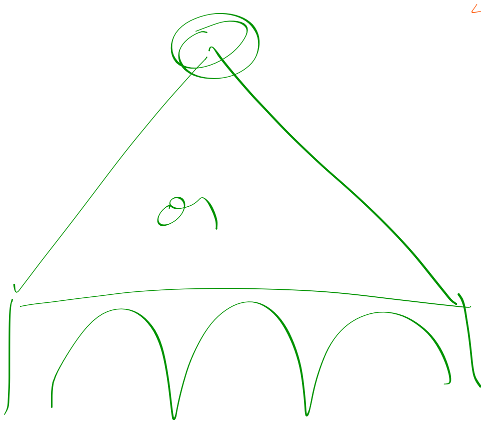
we have 2 cases

The Pont du Gard near Nîmes, France, is a Roman aqueduct. An observer in a hot-air balloon some distance away from the aqueduct determines that the angle of depression to each end is 54° and 71°, respectively. The closest end of the aqueduct is 270.0 m from the balloon. Calculate the length of the aqueduct to the nearest tenth of a metre.



$$\frac{a}{\sin 17} = \frac{270}{\sin 54^\circ}$$

$$\boxed{a = 97.6\text{m}}$$



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