2.4 Creating Equations in Kinematics

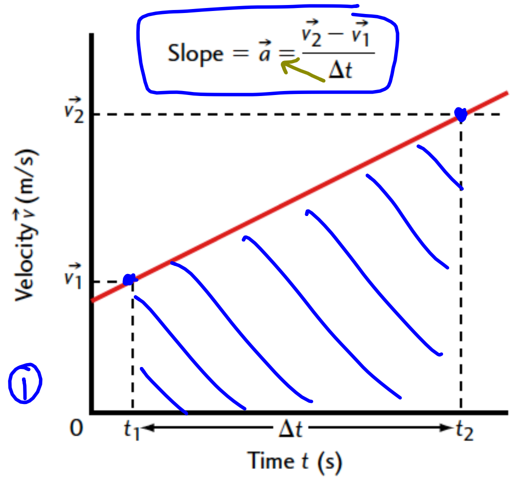
As we saw recently, slope of a $\vec{v}$-t graph yields acceleration so rearranging, we get...

solve for $\vec{v}_2$

$$\vec{a}\Delta t = \vec{v}_2 - \vec{v}_1 + \vec{v}_1$$

$+\vec{v}_1$

$$\boxed{\vec{v}_2 = \vec{v}_1 + \vec{a}\Delta t} \quad \text{equ}^n \ ①$$



In other words, if we know a body's initial velocity, acceleration and the time over which it is accelerating, we can find it's final velocity.

ex. A Boeing 737 spends 30 seconds accelerating before liftoff. If it's initial velocity is 27km/h and liftoff speed is 259.2 km/h,

a) what is it's acceleration?

given: $\vec{v}_1 = 27\text{ km/h}$, $\boxed{\vec{v}_1 = 7.5\text{ m/s}}$, $\boxed{\Delta t = 30\text{ s}}$

$\vec{v}_2 = 259.2\text{ km/h}$, $\boxed{\vec{v}_2 = 72\text{ m/s}}$

use equ^n ①

$$\vec{a} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t} = \frac{72 - 7.5}{30}$$

$$\boxed{\vec{a} = 2.15\text{ m/s}^2}$$ [forward]

with sig figs $\vec{a} = 2\text{ m/s}^2$

b) If it only spent 27 seconds accelerating, what would be the jet's final velocity?

given: $\vec{v}_1 = 7.5\text{ m/s}$, $\Delta t = 27\text{ s}$, $\vec{a} = 2.15\text{ m/s}^2$

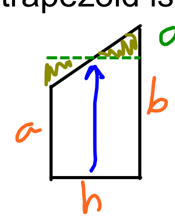
$$\vec{v}_2 = \vec{v}_1 + \vec{a}\Delta t = 7.5 + (2.15)(27)$$

$$\vec{v}_2 = 65.55\text{ m/s}$$

$$\boxed{\vec{v}_2 = 236\text{ km/h [forward]}}$$

with S.F. 2.4×10^2 km/h

If we find the area under a $\vec{v}$-t graph, we get displacement. Since the area of a trapezoid is



average height $= \boxed{\dfrac{a+b}{2}}$

$$A_{trap.} = h\left(\frac{a+b}{2}\right)$$

we get: $\boxed{\Delta\vec{d} = \frac{1}{2}(\vec{v}_1 + \vec{v}_2)\Delta t}$ equ^n ②

Note that these two equations were derived from straight-line $\vec{v}$-t graphs so they only work with constant acceleration.

NOW: if we substitute the first equation into the second, we get a third equation...

① $\vec{v}_2 = \vec{v}_1 + \vec{a}\Delta t$ ② $\Delta d = \frac{1}{2}(\vec{v}_1 + \vec{v}_2)\Delta t$

$$\Rightarrow \Delta d = \frac{1}{2}(\vec{v}_1 + \vec{v}_1 + \vec{a}\Delta t)\Delta t$$

$$\Delta d = \frac{1}{2}(2\vec{v}_1 + \vec{a}\Delta t)\Delta t$$

$$\Delta d = (\vec{v}_1 + \frac{1}{2}\vec{a}\Delta t)\Delta t$$

$$\boxed{\Delta d = \vec{v}_1\Delta t + \frac{1}{2}\vec{a}\Delta t^2}$$ equ^n ③

ex. A water balloon is thrown downward from the top of the school toward an unsuspecting eighth grader with an initial downward velocity of 2 m/s. How far does the balloon travel in the first 0.75 seconds of its trajectory?

given: $\vec{v}_1 = -2\text{ m/s}$

up = ⊕ $\Delta t = 0.75\text{ s}$

$\vec{a} = -9.8\text{ m/s}^2$

RTF = $\vec{\Delta d}$

$$\vec{\Delta d} = v_1\Delta t + \frac{1}{2}\vec{a}\Delta t^2$$

$$= (-2)(0.75) + \frac{1}{2}(-9.8)(0.75)^2$$

$$= -1.5 - 2.76$$

$$\boxed{\vec{\Delta d} = -4.26\text{ m}}$$

Similarly, if we rearrange equation one for $\vec{v}_1$ and substitute it into equation two, we get...

equ^n ④ $$\boxed{\Delta d = \vec{v}_2\Delta t - \frac{1}{2}\vec{a}\Delta t^2}$$

Notice that ③ and ④ are quadratic... so if you need to solve for Δt, this will be difficult without precalc 11.

Finally, if we isolate Δt from equation 1 and sub it into equation 2 we get...

① $\vec{v}_2 = \vec{v}_1 + \vec{a}\Delta t$

$\frac{\vec{v}_2 - \vec{v}_1}{\vec{a}} = \frac{\vec{a}\Delta t}{\vec{a}}$

$\frac{\vec{v}_2 - \vec{v}_1}{\vec{a}} = \Delta t$

② $\Delta \vec{d} = \frac{1}{2}(\vec{v}_1 + \vec{v}_2)\Delta t$

$2\vec{a}(\Delta \vec{d}) = \frac{1}{2}(\vec{v}_2 + \vec{v}_1)\left(\frac{\vec{v}_2 - \vec{v}_1}{\vec{a}}\right)(2\vec{a})$

$2\vec{a}\Delta\vec{d} = (\vec{v}_2 + \vec{v}_1)(\vec{v}_2 - \vec{v}_1)$

$2\vec{a}\Delta\vec{d} = \vec{v}_2^{\,2} - \vec{v}_1^{\,2}$

equ^n ⑤ $\boxed{\vec{v}_2^{\,2} = \vec{v}_1^{\,2} + 2\vec{a}\Delta\vec{d}}$

ex. At the beginning of a race a formula 1 race car maintains an initial pace velocity of 80 km/h. When the green flag is waved indicating the start of the race, Mario Andretti accelerates constantly over a distance of 500 m to attain a velocity of 220 km/h. What is Mario's acceleration?

given: $\vec{v}_1 = 80\ km/h$
$= 22.2\ m/s$
$\vec{v}_2 = 220\ km/h$
$= 61.1\ m/s$
$\Delta\vec{d} = 500\ m$
$= 0.5\ km$

RTF: $\vec{a} = ?$

Note: Forward is ⊕

equ^n ⑤: $\vec{v}_2^{\,2} = \vec{v}_1^{\,2} + 2\vec{a}\Delta\vec{d}$

$\frac{\vec{v}_2^{\,2} - \vec{v}_1^{\,2}}{2\Delta\vec{d}} = \frac{2\vec{a}\Delta\vec{d}}{2\Delta\vec{d}}$

$\frac{(61.\bar{1})^2 - (22.\bar{2})^2}{2(500)} = \vec{a}$

$\boxed{\vec{a} = 3.24\ m/s^2\ [\text{forward}]}$

* with sig figs $\vec{a} = 3\ m/s^2$ [forward]

REMEMBER: these equations only work for constant acceleration!!!

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