2.5 Solving Problems Involving Rates of Change

Sample problem:

Show that the minimum value for the function f(x) = x² + 4x - 21 occurs at x = -2

3 Strategies

Complete the $\square$

$y = (x^2 + 4x) - 21$

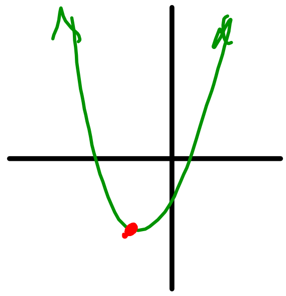
$= (x^2 + 4x + 4) - 4 - 21$

$= (x+2)^2 - 25$

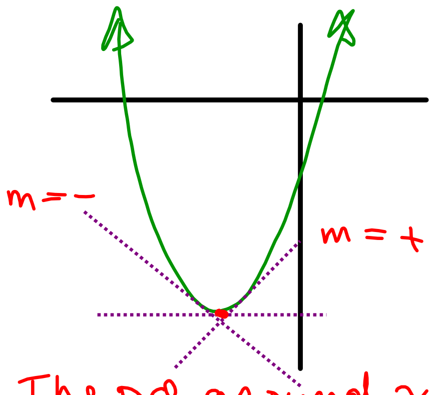
$\therefore$ vertex @ $(-2, -25)$

min value

graph it



slope of the tangent



The DQ around x = -2 goes from - to +

2 Definitions for the minimum of a function at x = a

The points around (a, f(a)) are greater than f(a)

* f(x) > f(a) for all x near a

* The function changes from decreasing to increasing
* IROC changes from - to +
* $IROC_{x=a} = 0$

The DQ is a good tool for this purpose:

→ find the DQ just to the left & just to the right of x = a . . .

DQ should change from - to +

Practice

For the following functions, use the DQ to show that the given value of x is a minimum.
[And then verify with a graphing calculator]

Example 1: $f(x) = x^3 - 6x^2 - 15x + 3$, at $x = 5$

$f(5) =$
$f(5.001) =$
$f(4.999) =$

Check DQ @ $a = 5$
$h = -0.001$

$DQ = \dfrac{f(a+h) - f(a)}{h}$

$= \dfrac{f(4.999) - f(5)}{-0.001}$

$= \dfrac{-96.999991 - (-97)}{-0.001}$

$= -0.009$

DQ @ $a = 5$
$h = 0.001$

$DQ = \dfrac{f(5.001) - f(5)}{0.001}$

$= 0.009001$

since DQ went from $-$ to $+$, there must be a min value @ $x = 5$

Example 2: $g(x) = 2 - 3\cos\left(2x - \dfrac{\pi}{4}\right)$, $at\ x = \dfrac{9\pi}{8}$

$g\left(\frac{9\pi}{8}\right) = -1$

$g\left(\frac{9\pi}{8} + 0.001\right) = -0.999994$

$g\left(\frac{9\pi}{8} - 0.001\right) = -0.999994$

$DQ = \dfrac{f\left(\frac{9\pi}{8} + 0.001\right) - f\left(\frac{9\pi}{8}\right)}{0.001} = 0.006$

$DQ = \dfrac{f\left(\frac{9\pi}{8} - 0.001\right) - f\left(\frac{9\pi}{8}\right)}{-0.001} = -0.006$

Same conclusion as in a)

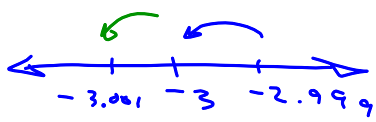
Example 3: $q(x) = -2x^2 - 12x + 9$, at $x = -3$

$f(-3.001) = 26.999998$
$f(-3) = 27$
$f(-2.999) = 26.999998$

$DQ = 0.002$

$DQ = \dfrac{f(-2.999) - f(-3)}{-0.001}$

$= -0.002$

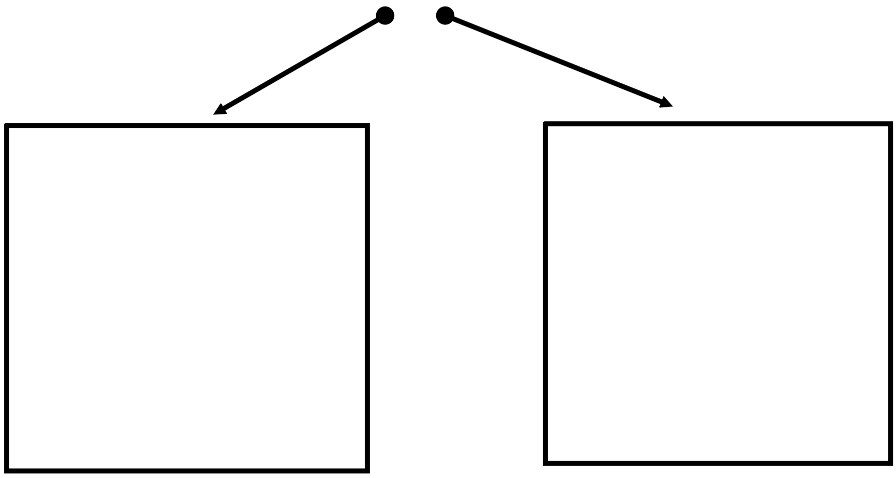


$\therefore$ slope @ $x = -3$ is zero

BUT the DQ goes from $+$ to $-$

So... $x = -3$ is a MAXIMUM

2 Definitions for the maximum of a function at $x = a$



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