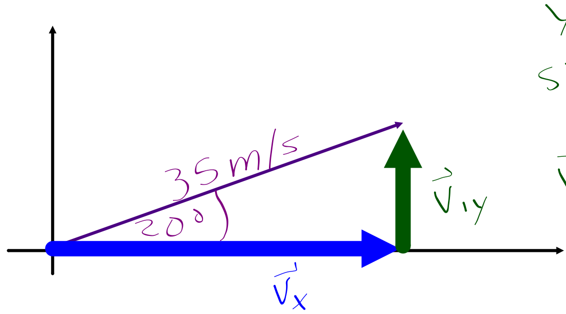
2.6 Projectile Motion (part 2)

If an object is launched at an angle, the flight path, or trajectory is affected by the launch angle. To determine how the projectile behaves, we must resolve the launch velocity into horizontal and vertical component velocities. This is when simple trig is useful. We must resolve a vector into it's vertical and horizontal components.

ex. A football is thrown with an initial velocity of 35 m/s at an angle of 20°.

a) What are the horizontal and vertical components of the velocity vector?



y-values

$\sin 20° = \frac{\vec{v}_{1y}}{35}$

$\vec{v}_{1y} = 35 \sin 20°$

$\boxed{\vec{v}_{1y} \doteq 12 \text{ m/s [up]}}$

$\boxed{\vec{v}_y = \vec{v}\sin\theta}$

x-values

$\cos 20° = \frac{\vec{v}_{1x}}{35}$

$\vec{v}_{1x} = 35 \cos 20°$

$\boxed{\vec{v}_{1x} \doteq 33 \text{ m/s [right]}}$

$\boxed{\vec{v}_x = \vec{v}\cos\theta}$

b) What is the football's max height

given: $\vec{v}_{1y} = 12$ m/s

$\vec{v}_{2y} = 0$ m/s

$\vec{a}_y = -9.8 \text{ m/s}^2$

$\Delta\vec{d}_y = ?$

(5) $\vec{v}_2^{\,2} = \vec{v}_1^{\,2} + 2\vec{a}\Delta\vec{d}$

$\frac{\cancel{\vec{v}_2^{\,2}}^{\,0} - \vec{v}_1^{\,2}}{2\vec{a}} = \Delta\vec{d}_y$

$\frac{-(12)^2}{2(-9.8)} = \Delta\vec{d}_y$

$\boxed{\Delta\vec{d}_y = 7.35 \text{ m [up]}}$

c) How far away does the football land?

RTF: Δt, then $\Delta\vec{d}_x$

must use vertical component

(1) $\cancel{\vec{v}_{2y}}^{\,0} = \vec{v}_{1y} + \vec{a}_y\Delta t$

$\frac{-\vec{v}_{1y}}{\vec{a}_y} = \Delta t$

$\Delta t = \frac{-(12)}{-9.8}$

$\boxed{\Delta t = 1.22 \text{ s}}$

for $\frac{1}{2}$ of the flight

$\Delta t = 2.44$ s

$\vec{v}_x = 33$ m/s

$\therefore \Delta\vec{d}_x = \vec{v}_x\Delta t$

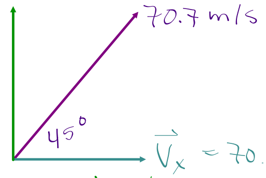
$= (33)(2.44)$

$\boxed{\Delta\vec{d}_x = 80.3 \text{ m [right]}}$

what do you think is the optimal launch angle? $\Rightarrow$ launch angle $= 45°$

s://www.physicsclassroom.com/Physics-Interactives/Vectors-and-Projectiles/Projectile-Simulator/Projectile-Simulator-Interactive

ex. What is the maximum distance a baseball can be hit horizontally if it's speed off the bat is 70.7 m/s



$\vec{V}_x = 70.7\cos 45° = 50\ m/s$

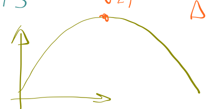
$\vec{V}_{1y} = 70.7\sin 45° = 50\ m/s$

$\rightarrow$ need Δt : use eqn ①

$\cancel{\vec{V}_{2y}}^{\,0} = \vec{V}_{1y} + \vec{a}_y \Delta t$

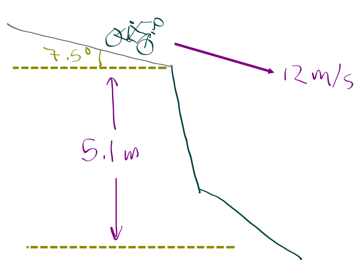
$\frac{-\vec{V}_{1y}}{\vec{a}_y} = \Delta t = \frac{-(50)}{-9.8} = 5.10\ s$

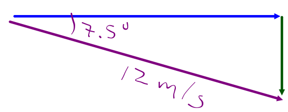
$\therefore$ total $\Delta t = 5.10 \times 2 = 10.20$



$\Delta d_x = \vec{V}_x \Delta t$
$= 50(10.20\ s)$
$= 510\ m$

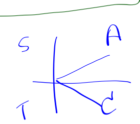
ex. A Redbull Rampage event has cyclists hitting a jump with a takeoff angle of -7.5° to the horizontal. The feature has a step-down distance of 5.1 metres and the riders know from experience that in order to be successful on this feature, an initial velocity of 12 m/s is required. How far, horizontally and ~~vertically~~ will the riders land?





$\vec{V}_{1y} = \vec{V}\sin\theta$
$= (12)\sin(-7.5)$
$\boxed{\vec{V}_{1y} = -1.566\ m/s}$

$\vec{V}_x = \vec{V}\cos\theta$
$= (12)(\cos -7.5°)$
$\boxed{\vec{V}_x = 11.90\ m/s}$



link x & y directions with Δt : given $\Rightarrow$ $\vec{V}_{1y} = -1.566\ m/s$, $\vec{a}_y = -9.8\ m/s^2$, $\Delta d_y = -5.1\ m$, $\Delta t = ?$

use ③ : $\Delta \vec{d} = \vec{V}_1 \Delta t + \frac{1}{2} a \Delta t^2$

$-5.1 = (-1.566)\Delta t + \frac{1}{2}(-9.8)\Delta t^2$

$4.9\Delta t^2 + 1.566\Delta t - 5.1 = 0 \quad (a = 4.9\ ;\ b = 1.566\ ;\ c = -5.1)$

Quadratic formula $\left.\right\}$ $\Delta t = \frac{-(1.566) \pm \sqrt{(1.566)^2 - 4(4.9)(-5.1)}}{2(4.9)}$

$= \frac{-1.566 \cancel{\pm} 10.120}{9.8}$

ignoring the negative Δt

$\boxed{\Delta t = 0.87285\ s}$

find $\Delta \vec{d}_x$ using $\vec{V}_x$ and Δt : $\Delta d_x = \vec{V}_x \Delta t$
$= (11.90)(0.87285)$
$\doteq 10.39\ m$ or
or $\boxed{1.0 \times 10^1\ m}$

$\therefore$ The rider travels about 10 m horizontally

October 21, 2021

Table 2.1
The Five Equations of Linear Kinematics

No.	Equation	$\Delta\vec{d}$	$\vec{a}$	$\vec{v}_2$	$\vec{v}_1$	Δt
1	$\vec{v}_2 = \vec{v}_1 + \vec{a}\Delta t$		✓	✓	✓	✓
2	$\Delta\vec{d} = \frac{1}{2}(\vec{v}_2 + \vec{v}_1)\Delta t$	✓		✓	✓	✓
3	$\Delta\vec{d} = \vec{v}_1\Delta t + \frac{1}{2}\vec{a}\Delta t^2$	✓	✓		✓	✓
4	$\Delta\vec{d} = \vec{v}_2\Delta t - \frac{1}{2}\vec{a}\Delta t^2$	✓	✓	✓		✓
5	$\vec{v}_2^2 = \vec{v}_1^2 + 2\vec{a}\Delta\vec{d}$	✓	✓	✓	✓	

not a vector

connect x & y directions

Practice: pg. 113 #29 -33

Quiz Tomorrow: 2.5 Equations of Motions

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Fig.3.16 Solving Projectile Problems