

3.1 & 3.2 Introduction to Polynomial Functions

Form of a polynomial function -

$$\begin{cases} \text{linear: } y = ax + b \\ \text{quadratic: } y = ax^2 + bx + c \\ \text{cubic: } y = ax^3 + bx^2 + cx + d \end{cases}$$

Polynomial of degree n

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \dots + a_1 x^1 + \underline{a_0}$$

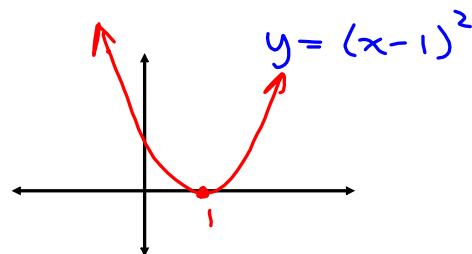
= Constant
= y-int.

Polynomial functions are ALWAYS

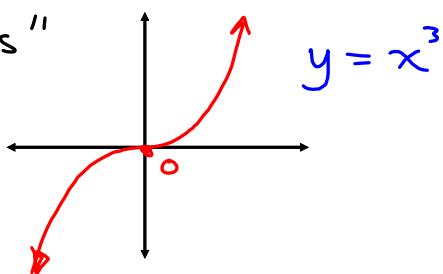
- degree one or more
- exponents \in Natural number set $\{1, 2, \dots\}$
- max # of turning points is one less than the degree
- domain $= \{x \in \mathbb{R}\}$ or $x \in (-\infty, \infty)$
- continuous

Roots

* double root
"bounces" off the x-axis

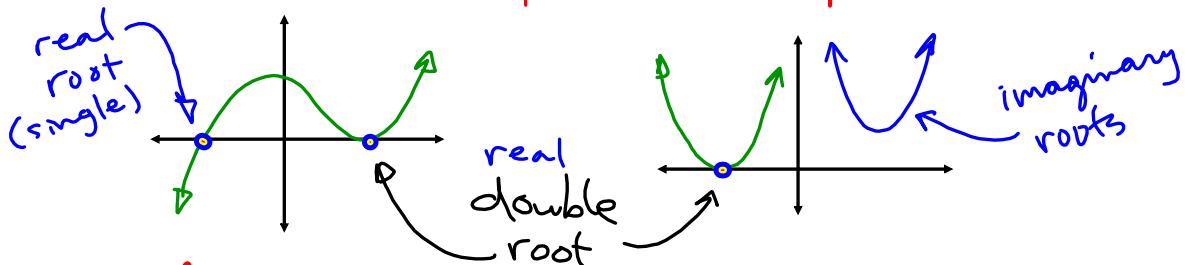


* triple root "skims" along the x-axis



X-Intercepts... Zeroes... (Roots, solutions...)

An n -degree function has n roots
 → these may be real, imaginary (complex), or "multiple" zeroes



* The minimum # of real roots for $y = P(x)$
 ODD $P(x)$ ⇒ one root
 EVEN $P(x)$ ⇒ none

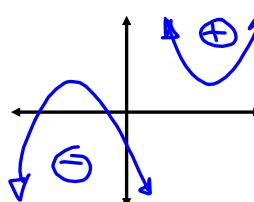
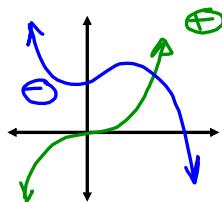
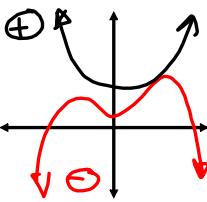
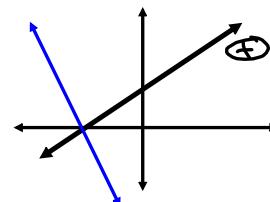
Turning Points

- EVEN $P(x)$ has, at most $n-1$ turning points (odd number)
- ODD $P(x)$ has, at most $n-1$ T.P.s (even number)

End Behaviour (for $\oplus P(x)$)

ODD → low to high [Q III → Q I]

EVEN → high to high [Q II → Q I]

	Quadratic	Cubic	Quartic	Linear
EQUATION	$f(x) = a_2x^2 + a_1x + a_0$	$f(x) = a_3x^3 + a_2x^2 + a_1x + a_0$	$f(x) = a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0$	$f(x) = a_1x + a_0$
Degree	2	3	4	
General Shape				
Turning Points	1	0, 2	3, 1	0
Y-intercept	a_0	a_0	a_0	a_0
End Behaviour	\oplus high \rightarrow high \ominus low \rightarrow low	\oplus low \rightarrow high \ominus high \rightarrow low	\oplus high \rightarrow high \ominus low \rightarrow low	\oplus low \rightarrow high \ominus high \rightarrow low

is determined by the sign of the leading coefficient

