

Chapter 3: Set Theory and Logic

1.1 Types of Sets and Set Notation

What categories can you use to sort your clothes?

Some definitions to get us started...

1. set: a collection of distinguishable objects or **elements**
ex. *whole numbers* = $\{0, 1, 2, 3, \dots\} = W$
2. subset: a set whose elements **all belong** to **another** set
ex. *odd digits* = $\{1, 3, 5, 7, \dots\} \Rightarrow O \subset W$
3. universal set: a set of **all** the elements under consideration for a particular context
ex. *Z (integers)* = $\{\dots, -2, -1, 0, 1, 2, \dots\}$
4. complement: all the elements of a universal set that **do not** belong to a subset of it
ex. *O'* (the complement of O) = $\{0, 2, 4, \dots\}$
5. empty set: a set with **no elements**
ex. *odd digits divisible by 2* = $\{\}$ or \emptyset
6. disjoint: two or more sets having **no elements** in **common**
ex. *the sets of even and odd numbers*

Subset notation

Communication Notation

The following is a summary of notation introduced so far.

Sets are defined using brackets. For example, to define the universal set of the numbers 1, 2, and 3, list its elements:

$$U = \{1, 2, 3\}$$

To define the set A that has the numbers 1 and 2 as elements:

$$A = \{1, 2\}$$

All elements of A are also elements of U , so A is a subset of U :

$$A \subset U$$

The set A' , the complement of A , can be defined as:

$$A' = \{3\}$$

To define the set B , a subset of U that contains the number 4:

$$B = \{\} \quad \text{or} \quad B = \emptyset$$

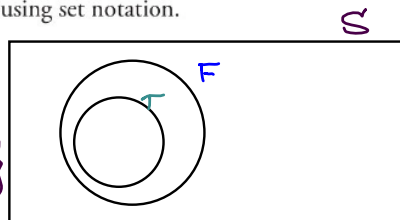
$$B \subset U$$

Investigate pg. 146 together

EXAMPLE 1 | Sorting numbers using set notation and a Venn diagram

- a) Indicate the multiples of 5 and 10, from 1 to 500, using set notation.
List any subsets.

- b) Represent the sets and subsets in a Venn diagram.



a) $S = \{1, 2, 3, \dots, 499, 500\}$

$S = \{x \mid 1 \leq x \leq 500, x \in \mathbb{N}\}$

$F = \{5, 10, 15, \dots, 495, 500\}$

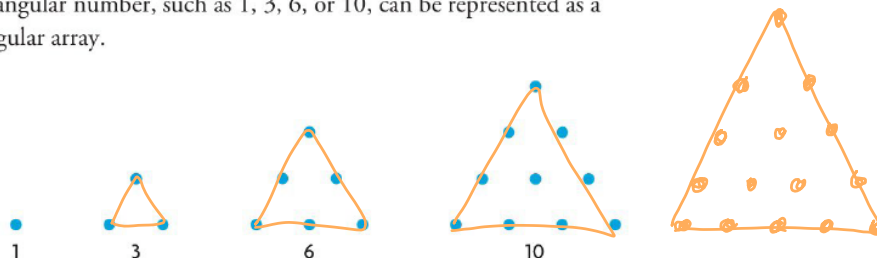
$F = \{f \mid f = 5x, 1 \leq x \leq 100, x \in \mathbb{N}\}$

$T = \{t \mid t = 10x, 1 \leq x \leq 50, x \in \mathbb{N}\}$

$T \subset F \subset S$

EXAMPLE 2 | Determining the number of elements in sets

A triangular number, such as 1, 3, 6, or 10, can be represented as a triangular array.



- a) Determine a pattern you can use to determine any triangular number.
b) Determine how many natural numbers from 1 to 100 are
i) even and triangular,
ii) odd and triangular, and
iii) not triangular.
c) How many numbers are triangular?

if $n = 4$, $t_4 =$ the sum of the 1st n natural numbers

a) $1, 1+2, 1+2+3, 1+2+3+4, \dots$

$1, 3, 6, 10, \dots$

$T = \{\text{triangular}\} = \{1, 3, 6, 10, 15, 21, 28, 36, 45, 55, 66, 78, 91, 105\}$

b) i- $E = \{\text{even triangular}\}$

$= \{6, 10, 28, 36, 66, 78\} \Rightarrow n\{E\} = 6$

ii) $O = \{\text{odd triangular}\}$

$= \{1, 3, 15, 21, 45, 55, 91\} \Rightarrow n\{O\} = n\{E\} = 7$

iii)- since $n\{T\} = 13$

$n\{T'\} = 100 - 13 = 87$

c) $n\{T\} = 13$

EXAMPLE 3

Describing the relationships between sets

Alden and Connie rescue homeless animals and advertise in the local newspaper to find homes for the animals. They are setting up a web page to help them advertise the animals that are available. They currently have dogs, cats, rabbits, ferrets, parrots, lovebirds, macaws, iguanas, and snakes.

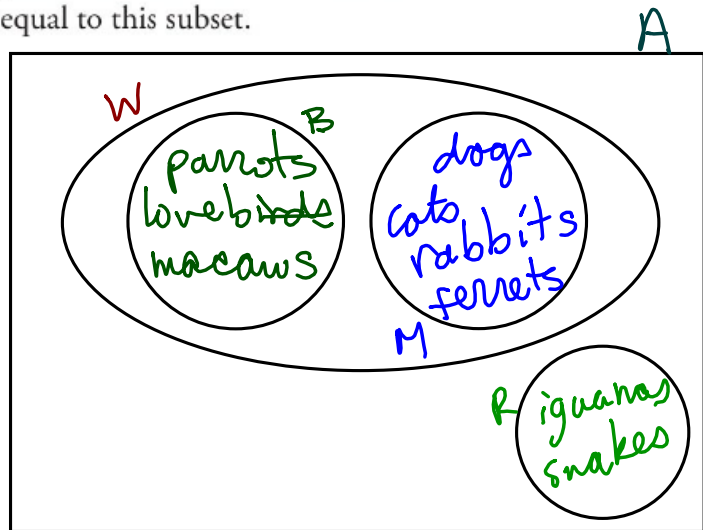
- Design a way to organize the animals on the web page. Represent your organization using a Venn diagram.
- Name any disjoint sets.
- Show which sets are subsets of one another using set notation.
- Alden said that the set of fur-bearing animals could form one subset. Name another set of animals that is equal to this subset.

- a) $A = \{\text{all animals}\}$
 $B = \{\text{birds}\}$
 $M = \{\text{mammals}\}$
 $R = \{\text{reptiles}\}$
 $W = \{\text{warmblooded}\}$
 $C = \{\text{coldblooded}\}$
- Same →

- b) disjoint: B and R
 B and M
 W and C
 R and M

- c) $R \subset A, M \subset A, B \subset A,$
 $W \subset A, B \subset W, M \subset W$

- d) if $F = \{\text{fur-bearing}\}$,
 then $F = M$



In Summary

Key Ideas

- You can represent a set of elements by:
 - listing the elements; for example, $A = \{1, 2, 3, 4, 5\}$
 - using words or a sentence; for example, $A = \{\text{all integers greater than 0 and less than 6}\}$
 - using set notation; for example, $A = \{x \mid 0 < x < 6, x \in \mathbb{I}\}$
- You can show how sets and their subsets are related using Venn diagrams. Venn diagrams do not usually show the relative sizes of the sets.
- You can often separate a universal set into subsets, in more than one correct way.

Need to Know

- Sets are equal if they contain exactly the same elements, even if the elements are listed in different orders.
- You may not be able to count all the elements in a very large or infinite set, such as the set of real numbers.
- The sum of the number of elements in a set and its complement is equal to the number of elements in the universal set:
 $n(A) + n(A') = n(U)$
- When two sets A and B are disjoint,
 $n(A \text{ or } B) = n(A) + n(B)$

