3.2 Perfect Squares, Perfect Cubes and Their Roots

rational # (fraction)

*If a number can be represented as the area of a square whereby the sides of the square are whole numbers, we call the number a perfect square.

Ex. $\sqrt{81} = 9$ $\therefore$ 81 is a perfect square ex// $\sqrt{\frac{81}{16}} = \frac{9}{4}$

*100 is a perfect square but 100 = $10 \cdot 10 = 2 \cdot 5 \cdot 2 \cdot 5 = 2^2 \cdot 5^2$

Note: all prime factors of a perfect square have even powers.

Ex. 324 =

324 = 9 · 36; 9 = 3 · 3; 36 = 4 · 9; 4 = 2 · 2; 9 = 3 · 3

$3 + 2 + 4 = 9$

$\therefore 324 = 2^2 \cdot 3^4$

$\sqrt{324} = 2 \cdot 3^2$

$= 18$

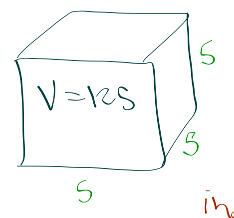
* $\sqrt{x} = x^{1/2}$

$\sqrt{x^6} = x^3$

*If a number can be represented as the volume of a cube whereby the sides of the cube are whole numbers, we call the number a perfect cube.

or rational

ex. consider 125 = 5^3



radical

Note: all prime factors of a perfect cube have powers that are multiples of 3

$\sqrt[3]{125} = 5$

index

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ex. $\sqrt[3]{1728}$

1728 = 192 · 9; 192 = 3 · 64; 64 = 8 · 8; 8 = 2^3; 8 = 2^3; 9 = 3 · 3

$\therefore 1728 = 2^{6 \div 3} \cdot 3^{3 \div 3}$

$\sqrt[3]{1728} = 2^2 \cdot 3$

$= 12$

ex. Make up a number that is a perfect square AND a perfect cube at the same time

$3^6 = 729$ test $\sqrt{729} = 27$

$\sqrt[3]{729} = 9$

or $2^6 \cdot 5^6 = 1\,000\,000$

$\sqrt{1\,000\,000}$

$= 1000$

$\sqrt[3]{1\,000\,000} = 100$

Homefun: Pg. 146 #(4-18)ace