p 158 # 9b

vertex $\Rightarrow (0,-6) \therefore p=0, q=-6$

$y = a(x-0)^2 - 6$

$y = ax^2 - 6$ but passing through $(3, 21)$

$21 = a(3)^2 - 6$

$\frac{27}{9} = \frac{9a}{9} \Rightarrow a = 3 \quad \therefore \boxed{y = 3x^2 - 6}$

3.3 Completing the Square

homefun: page 192 #1-4, 8, 9, 14, 16, 17, 19, 22, ~~27~~, 29-30

Recap: a) $(x+4)^2$ b) $(x-3)^2$ c) $(x+a)^2$

a) $= (x+4)(x+4)$
$= x^2 + 4x + 4x + 16$
$= x^2 + 8x + 16$

b) $= (x-3)(x-3)$
$= x^2 - 6x + 9$

c) $= (x+a)(x+a)$
$= x^2 + 2ax + a^2$

Are these perfect squares? How else could you write them?

a) $y = x^2 + 2x + 1$
$= (x+1)(x+1)$
$= (x+1)^2$

b) $x^2 - 6x + 9$
$= (x-3)^2$

What numbers could complete the square?

a) $y = x^2 + 10x + ?$
$= x^2 + 10x + 25$
$= (x+5)^2$

b) $x^2 + ?x + 36$
$= x^2 \pm 12x + 36$
$= (x+6)^2$
or $= (x-6)^2$

a) $y = x^2 + 7x + ?$
$\left(\frac{7}{2}\right)^2 = \frac{49}{4}$
$y = x^2 + 7x + \frac{49}{4}$
$y = \left(x + \frac{7}{2}\right)^2$

The process of finding the "?" to make a perfect square trinomial can also be used to change an expression into vertex form.
In this case, however, we can't just make the number we want magically appear, so we must use a little trick.........

Example 1: Write the quadratic relation $y = x^2 - 8x - 2$ in vertex form.

$y = (x^2 - 8x) - 2$
$y = (x^2 - 8x + 16 - 16) - 2$
$y = (x^2 - 8x + 16) - 16 - 2$
$y = (x-4)^2 - 16 - 2$
$\boxed{y = (x-4)^2 - 18}$ vertex form

STEPS...

1. group 1st and 2nd term
2. add and subtract $(b/2)^2$
3. push out the negative constant (-16)
4. write the trinomial as $(x \pm a)^2$
5. combine the outside constants

Example 2: Write the quadratic relation $y = 3x^2 + 18x + 20$ in vertex form.

$y = (3x^2 + 18x) + 20$
$y = 3(x^2 + 6x) + 20$
$y = 3(x^2 + 6x + 9 - 9) + 20$
$y = 3(x^2 + 6x + 9) - 9(3) + 20$
$y = 3(x+3)^2 - 27 + 20$
$\boxed{y = 3(x+3)^2 - 7}$

STEPS...

1. Same as above
2. factor out "a" from brackets
3. add and subtract $(b/2)^2$
4. Same as 3. above but multiply by a
5. write as $a(x \pm p)^2 + q$

Example 3: Write the quadratic relation $y = 2x^2 + 9x + 5$ in vertex form.

$$y = (2x^2 + 9x) + 5$$
$$y = 2\left(x^2 + \tfrac{9}{2}x\right) + 5$$
$$y = 2\left(x^2 + \tfrac{9}{2}x + \tfrac{81}{16} - \tfrac{81}{16}\right) + 5$$
$$y = 2\left(x^2 + \tfrac{9}{2}x + \tfrac{81}{16}\right) - \tfrac{81}{\cancel{16}\,8}\cancel{(2)} + \tfrac{5 \times 8}{1 \times 8}$$
$$y = 2\left(x + \tfrac{9}{4}\right)^2 - \tfrac{81}{8} + \tfrac{40}{8}$$

$$\left(\tfrac{9}{2} \div 2\right)^2$$
$$\left(\tfrac{9}{2} \times \tfrac{1}{2}\right)^2$$
$$\left(\tfrac{9}{4}\right)^2 = \tfrac{81}{16}$$

$$\Rightarrow \boxed{y = 2\left(x + \tfrac{9}{4}\right)^2 - \tfrac{41}{8}}$$

So what's the point?

$\therefore$ vertex $(-9/4, -41/8)$

Well, now we have ANOTHER way (and maybe a weeeee bit faster) to get a expression into vertex form... which is useful for finding the vertex, and for solving.

Try this one:

Gumdrop Joe slips on the slippery FH parking lot and falls to the ground. His hat however flies into the air with the greatest of ease. The height of his hat is described by the equation $h = -5t^2 + 20t + 1$

(a) What is the maximum height of the hat? $\Rightarrow$ vertex ?

~~(b) When will the hat hit the ground?~~

$$h = (-5t^2 + 20t) + 1$$
$$h = -5\left(t^2 + \tfrac{20t}{-5}\right) + 1$$
$$h = -5(t^2 - 4t) + 1$$
$$h = -5(t^2 - 4t + 4) - 4(-5) + 1$$
$$h = -5(t - 2)^2 + 20 + 1$$
$$\boxed{h = -5(t-2)^2 + 21}$$

$$\left(\tfrac{-4}{2}\right)^2 = 4$$

$\therefore$ Since the parabola opens down $(a = -5)$ and the vertex is @ $(2, 21)$, the max height is 21 m