3.3 General Form to Vertex Form

To convert from general [$y = ax^2 + bx + c$] to vertex form [$y = a(x - p)^2 + q$], we must pass through the equation for the axis of symmetry:

$$x = \frac{-b}{2a}$$

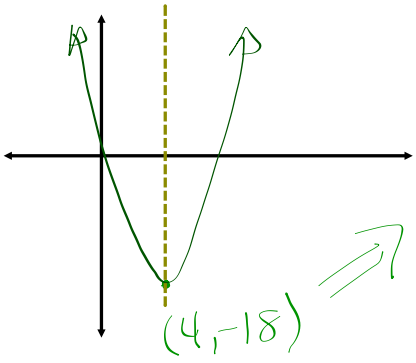
Example 1: Write the quadratic relation $y = x^2 - 8x - 2$ in vertex form.

$x = \frac{-(-8)}{2(1)} = 4$

$y = (4)^2 - 8(4) - 2$

$y = 16 - 32 - 2$

$y = -18$



$p = 4$

$q = -18$

And since a is the same in both form and $a = 1 \Rightarrow$ $y = (x-4)^2 - 18$

Example 2: Convert $y = 3x^2 + 18x + 20$ to vertex form.

$x_v = \frac{-18}{2(3)} = -3$

$y_v = 3(-3)^2 + 18(-3) + 20$

$= 27 - 54 + 20$

$= -7$

$a = 3$

$p = -3$

$q = -7$

$\therefore y = 3(x - (-3))^2 - 7$

$y = 3(x+3)^2 - 7$

Example 3: Write the quadratic relation $y = 2x^2 + 9x + 5$ in vertex form.

$x = \frac{-9}{2(2)} = -\frac{9}{4} = -2.25$

$y = 2\left(-\frac{9}{4}\right)^2 + 9\left(-\frac{9}{4}\right) + 5$

$= 2\left(\frac{81}{16}\right) - \frac{81}{4} + 5$ (×2, ×8, ×8)

$= \frac{81}{8} - \frac{162}{8} + \frac{40}{8}$

$= -\frac{41}{8}$

$a = 2$

$p = -\frac{9}{4}$

$q = -\frac{41}{8}$

$\therefore y = 2\left(x + \frac{9}{4}\right)^2 - \frac{41}{8}$

So what's the point?

Well, now we have ANOTHER way (and maybe a weeeee bit faster) to get an expression into vertex form... which is useful for finding the vertex, and for solving.

Try this one:

Gumdrop Joe slips on the slippery FH parking lot and falls to the ground. His hat however flies into the air with the greatest of ease. The height of his hat is described by the equation $h = -5t^2 + 20t + 1$

(a) What is the maximum height of the hat?

(b) When will the hat hit the ground?

a) $t = \frac{-b}{2a}$

$= \frac{-(20)}{2(-5)}$

$t = 2$

$\Rightarrow h = -5(2)^2 + 20(2) + 1$

$= -20 + 40 + 1$

$h = 21$

∴ since the vertex is at (2, 21), the hat's max height is 21 ft @ 2 seconds AND $h = -5(t-2)^2 + 21$ ⇒ vertex form

b) to get the x-int we need to set $h = 0$

$0 = -5(t-2)^2 + 21$

$-21 = -5(t-2)^2$

$\frac{-21}{-5} = \frac{-5(t-2)^2}{-5}$

$\pm\sqrt{4.2} = \sqrt{(t-2)^2}$

$\pm 2.05 = t - 2$

$2.05 = t - 2$

$4.05 = t$

also $-2.05 = t - 2$

$-0.05 = t$

meaningless in real life but a zero nonetheless!

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