# 3.3 Intersection and Union of Two Sets

intersection: the set of elements that are common to two or more sets

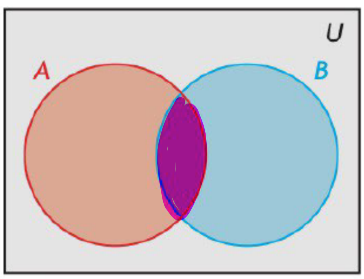
ex. if A = {1, 2, 3} and B = {3, 4, 5} then $A \cap B$ = {3}

union: the set of all elements in two or more sets

ex. if A = {1, 2, 3} and B = {3, 4, 5} then $A \cup B$ = {1, 2, 3, 4, 5}

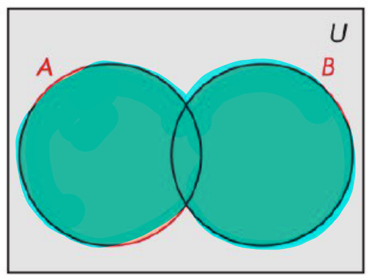
## Communication | *Notation*

In set notation, $A \cap B$ is read as "intersection of *A* and *B*." It denotes the elements that are common to *A* and *B*. The intersection is the region where the two sets overlap in the Venn diagram below.



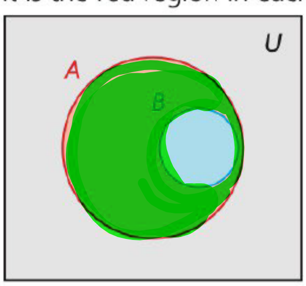

$A \cap B$

$A \cup B$ is read as "union of *A* and *B*." It denotes all elements that belong to at least one of *A* or *B*. The union is the red region in the Venn diagram below.

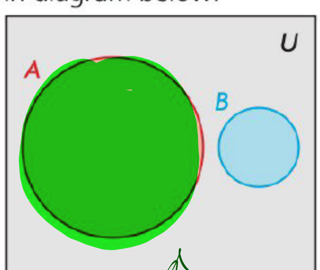



$A \cup B$

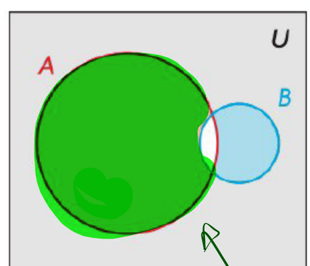
$A \setminus B$ is read as "*A* minus *B*." It denotes the set of elements that are in set *A* but not in set *B*. It is the red region in each Venn diagram below.




$A \setminus B$ when $B \subset A$




$A \setminus B$ when they are disjoint



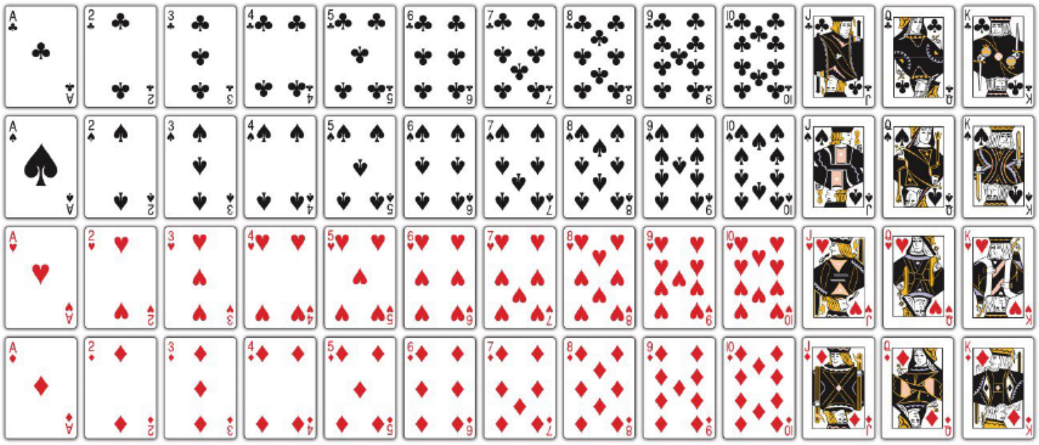

$A \setminus B$ when they intersect

A and B are mutually exclusive

not mutually exclusive

**EXAMPLE 1** Determining the union and intersection of disjoint sets

If you draw a card at random from a standard deck of cards, you will draw a card from one of four suits: clubs ($C$), spades ($S$), hearts ($H$), or diamonds ($D$).



**a)** Describe sets $C$, $S$, $H$, and $D$, and the universal set $U$ for this situation.
**b)** Determine $n(C)$, $n(S)$, $n(H)$, $n(D)$, and $n(U)$.
**c)** Describe the union of $S$ and $H$. Determine $n(S \cup H)$.
**d)** Describe the intersection of $S$ and $H$. Determine $n(S \cap H)$.
**e)** Determine whether the events that are described by sets $S$ and $H$ are mutually exclusive, and whether sets $S$ and $H$ are disjoint.
**f)** Describe the complement of $S \cup H$.

a) $U = \{\text{all 52 cards}\}$

$C = \{\text{clubs}\}$

$S = \{\text{spades}\}$

$H = \{\text{hearts}\}$

$D = \{\text{diamonds}\}$

b) $n(U) = 52$

$n(S) = n(C) = n(D) = n(H) = 13$

c) all spades and hearts

$n(S \cup H) = 26$

d) Spades that are also hearts

$n(S \cap H) = \{\ \} = \varnothing$

e) since S and H are disjoint they are mutually exclusive

$\cup$ = "OR" $\quad \cap$ = "AND"

f) $(S \cup H)' = (D \cup C)$

"all the cards that are diamonds OR clubs"

### *Your Turn*

Petra thinks that $n(S) + n(H) = n(S \cup H)$. Is she correct? Explain. ✓

$13 + 13 = 26 \Rightarrow (S \cup H)$ is all hearts OR spades

but $n(S) + n(H) \neq n(S \cap H)$

$13 + 13 \neq \varnothing$

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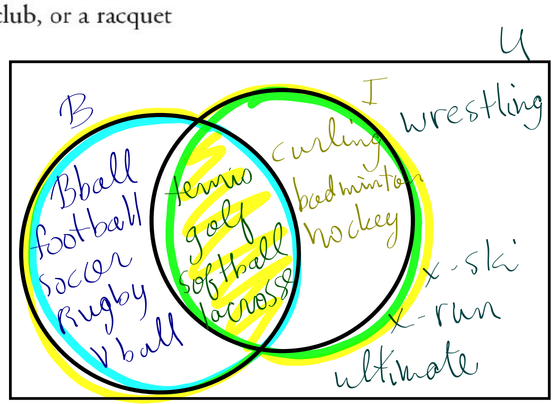
**EXAMPLE 2** Determining the number of elements in a set using a formula

The athletics department at a large high school offers 16 different sports:

| | | |
|---|---|---|
| badminton | hockey | tennis |
| basketball | lacrosse | ultimate |
| cross-country running | rugby | volleyball |
| curling | cross-country skiing | wrestling |
| football | soccer | |
| golf | softball | |

Determine the number of sports that require the following types of equipment:

**a)** a ball and an implement, such as a stick, a club, or a racquet
**b)** only a ball
**c)** an implement but not a ball
**d)** either a ball or an implement
**e)** neither a ball nor an implement




$U = \{\text{all sports offered}\}$

$B = \{\text{ball only}\}$

$I = \{\text{implement}\}$

a) $n\{B \cap I\} = 4$

b) $n\{B \setminus I\} = n\{B\} - n\{B \cap I\}$
$= 9 - 4$
$= 5$

c) $n\{I \setminus B\}$
$= n\{I\} - n\{B \cap I\}$
$= 7 - 4$
$= 3$

d) $n(B \cup I) = n(B) + n(I) - n(B \cap I) = 9 + 7 - 4 = 12$

e) $n((B \cup I)') = n(U) - n(B \cup I)$
$= 16 - 12 = 4$

Principle of Inclusion and Exclusion:

The number of elements in the union of 2 sets is the number of elements in each minus the number in both

$$n(A \cup B) = \underbrace{n(A) + n(B)}_{\text{the sum of both}} - \underbrace{n(A \cap B)}_{\text{overlap}}$$

$$n(A \cap B) = n(A) + n(B) - n(A \cup B)$$

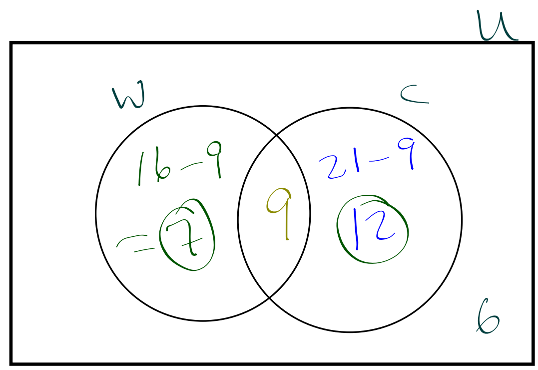
**EXAMPLE 3** Determining the number of elements in a set by reasoning

Jamaal surveyed 34 people at his gym. He learned that 16 people do weight training three times a week, 21 people do cardio training three times a week, and 6 people train fewer than three times a week.
How can Jamaal interpret his results?

43 − 34 = 9
must do both

number surveyed = 34
sum of survey data = 16 + 21 + 6 = 43
∴ there must be overlap b/w weights and cardio

U = {all surveyed}
W = {weights}
C = {cardio}

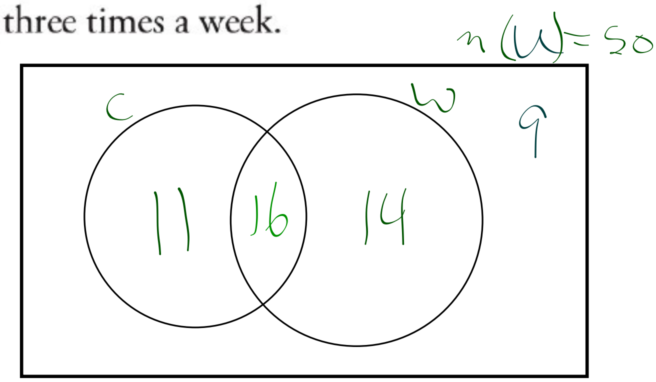



## *Your Turn*

Jamaal surveyed 50 other gym members. Of these members, 9 train fewer than three times a week, 11 only do cardio training three times a week, and 16 do both cardio and weight training three times a week. Determine how many of these members do weight training three times a week.

$n(C \cup W) = 50 - 9 = 41$

$41 - 11 - 16 = 14$




Example 4 pg. 169 together

Read Key Ideas pg. 171

Homework: pg. 172 # 5, 6, 7, 9, 10, 11*, 12, 16, 17, 18