MHF 4U name: ___________________

Zeroes of Polynomial Functions

Based on your previous knowledge of linear and quadratic functions, and what we have learned about the characteristics of polynomial functions, fill in the table below:

Function	Degree	as $x \to \infty, y \to$	as $x \to -\infty, y \to$	The x-intercept(s)
$f(x) = x + 3$	1	∞	$-\infty$	-3
$f(x) = -x - 5$	1	$-\infty$	∞	-5
$f(x) = (x+3)(x-4)$	2	∞	∞	-3 & 4
$f(x) = x(x-2)$	2	∞	∞	0 & 2
$f(x) = -2(x+10)(x-3)$	2	$-\infty$	$-\infty$	-10 & 3
$f(x) = (x+6)^2$	2	∞	∞	-6 (double root)

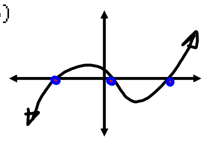
Now, using the information in the table above formulate answers for the next table.

Function	Degree	as $x \to \infty, y \to$	as $x \to -\infty, y \to$	The x-intercept(s)
$f(x) = (x+3)(x+2)(x-1)$	3	∞	$-\infty$	$-3, -2, 1$
$f(x) = -3(x-1)(x+4)(x-5)$	3	$-\infty$	∞	$1, -4, 5$
$f(x) = (x+11)^2(x-2)$	3	∞	$-\infty$	$-11, 2$
$f(x) = x(x-2)(x-5)$	3	∞	$-\infty$	$0, 2, 5$
$f(x) = -(x-6)^3$	3	$-\infty$	∞	6
$f(x) = (x+6)(x-2)(x-1)(x-9)$	4	∞	∞	$-6, 2, 1, 9$
$f(x) = (x+6)^2(x-2)(x+1)$	4	∞	∞	$-6, 2, -1$
$f(x) = -5.2(x+6)^2(x+2)^2$	4	$-\infty$	$-\infty$	$-6, -2$
$f(x) = x(x+7)^3$	4	∞	∞	$0, -7$

Use your graphing calculator to verify your work (not ALL of them, just the ones you're not sure of) and then make sketchs of the following types of zeroes. Make sure you use pencil!

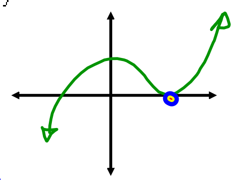
Single Zero (example: $x = -2$ for $f(x) = (x+3)(x+2)(x-1)$ is a single zero)

$\to$ crosses the x-axis

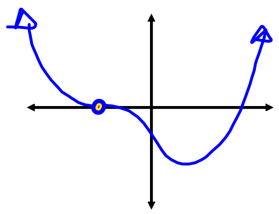


Double Zero (example: $x = -2$ for $f(x) = (x+3)(x+2)^2$ is a double zero)

$\to$ bounces off x-axis



Triple Zero (example: $x = -2$ for $f(x) = (x+2)^3(x-1)$ is a triple zero)



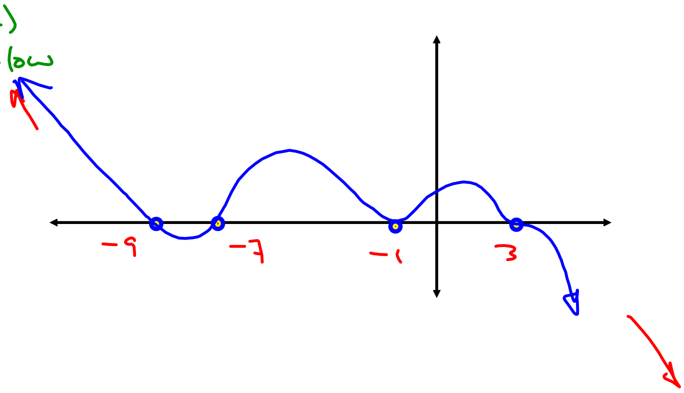
"skims" along the x-axis

$\to$ at the zero, IROC = 0

ex/ graph $f(x) = -(x+1)^2(x-3)^3(x+7)(x+9)$

degree 7 (-ve)

E.B. $\Rightarrow$ high $\to$ low



Zeroes of Polynomial Functions

When written in factored form... $f(x) = a(x-s)(x-t)(x-r)\ldots$

we get ① zeroes
② end behaviour (sign of a and degree)
③ type of x-intercepts

Family of functions...
a set of functions sharing the same zeroes

The sign of "a" determines the sign of the leading coefficient...

$f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots$

For odd degree polynomial functions

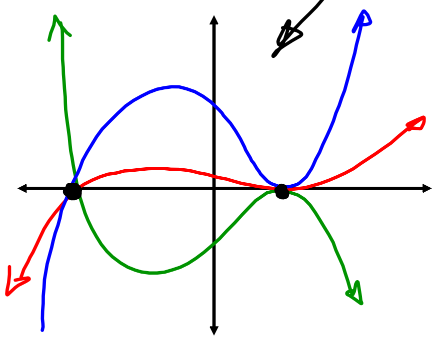
if $a > 0$: low to high

if $a < 0$: high to low

For even degree polynomial functions

if $a > 0$: high to high

if $a < 0$: low to low



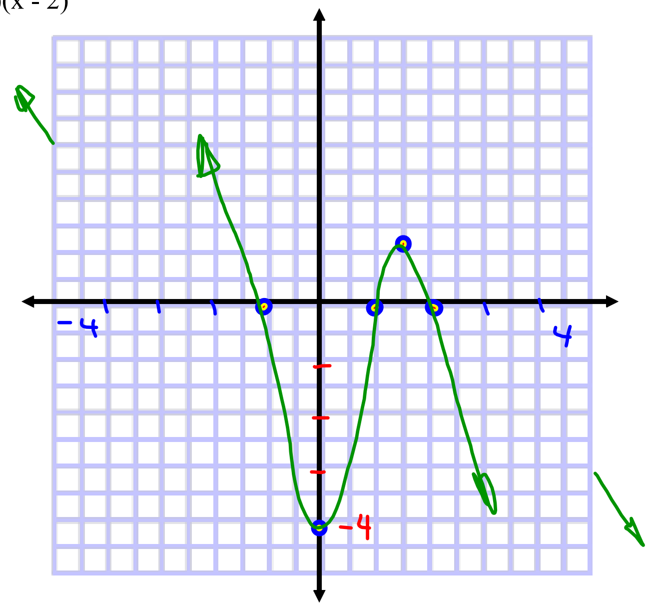
Example 1: Sketch a graph of $f(x) = -2(x - 1)(x + 1)(x - 2)$

y - intercept

$f(0) = -2(-1)(1)(-2)$
$= -4$

x - intercepts $1, -1, 2$

EB = high to low



test points $x = 1.5$

since this is b/w roots

$f(1.5) = -2(1.5-1)(1.5+1)(1.5-2)$
$= -2(0.5)(2.5)(-0.5)$
$= 1.25$

Example 2: A quartic function has zeroes at -1, -2, 2 and 4. It also has a y-intercept at 8.
Determine the functions equation.

$f(x) = a(x+1)(x+2)(x-2)(x-4)$

but I know a pt on $f(x) \ldots (0, 8)$

$\therefore \quad 8 = a(0+1)(0+2)(0-2)(0-4)$

$\frac{8}{16} = \frac{a(16)}{16}$

$a = \frac{1}{2}$

$\therefore f(x) = \frac{1}{2}(x+1)(x+2)(x-2)(x-4)$

Example 3: Sketch the graph of $f(x) = x^4 + 2x^3$

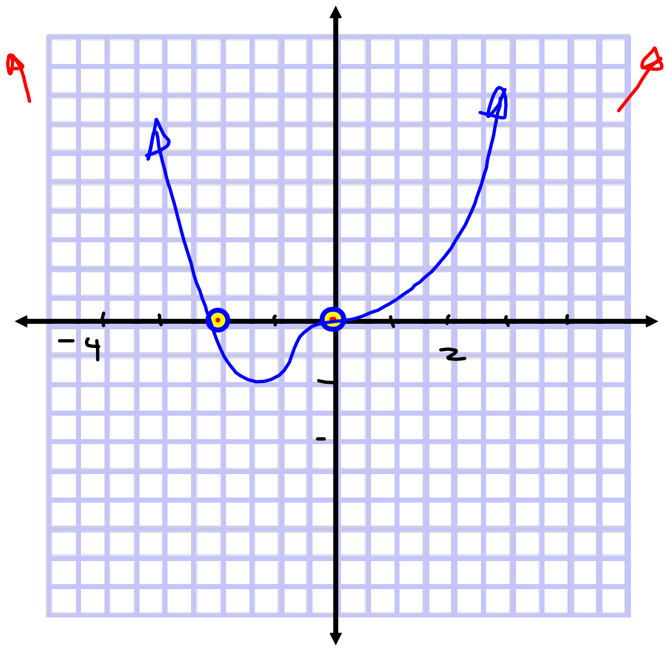
$f(x) = x^3(x+2)$

triple zero

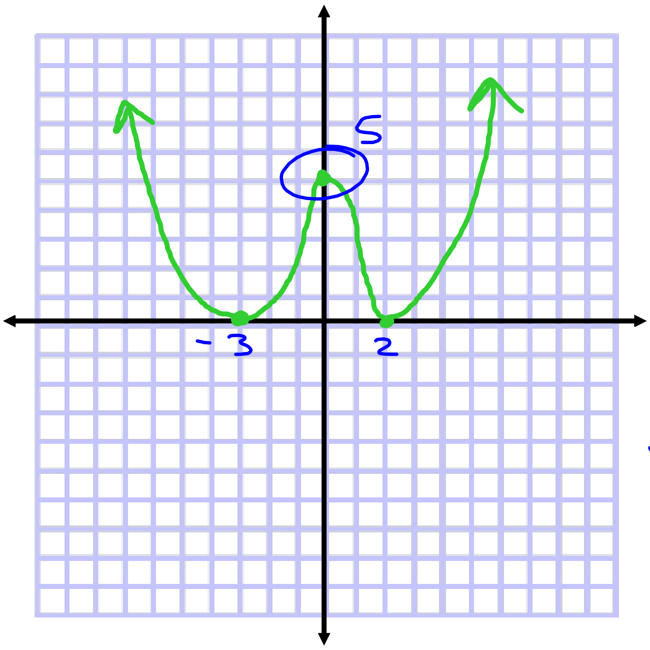
single root

EB : high to high

I could test $x = -1$
or $x = -1.5$



Example 4: Find the equation of the graph shown:



$f(x) = a(x+3)^2(x-2)^2$

but $(0,5)$ is on $f(x)$

$5 = a(0+3)^2(0-2)^2$

$\frac{5}{36} = \frac{a(3\cancel{6})}{\cancel{36}}$

$\therefore f(x) = \frac{5}{36}(x+3)^2(x-2)^2$

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