3.4 Applications of Set Theory

Explore pg. 179

EXAMPLE 1　　Solving a puzzle using the Principle of Exclusion and Inclusion

Use the following clues to answer the questions below:
- 28 children have a dog, a cat, or a bird.
- 13 children have a dog.
- 13 children have a cat.
- 13 children have a bird.
- 4 children have only a dog and a cat.
- 3 children have only a dog and a bird.
- 2 children have only a cat and a bird.
- No child has two of each type of pet.

a) How many children have a cat, a dog, and a bird?
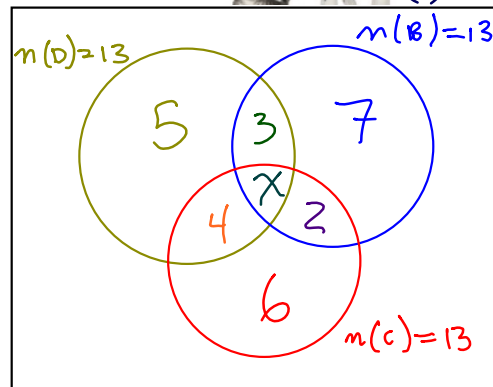**b)** How many children have only one pet?

$P = \{$ kids with pets $\}$

$B = \{$ birds $\}$

$C = \{$ cats $\}$

$D = \{$ dogs $\}$

$n(P) = 28$

$n(B) = 13$

$n(D) = 13$

$5 \quad 3 \quad 7$
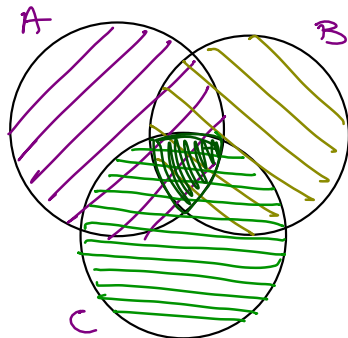
x

$4 \quad 2$

6

$n(c) = 13$

With some guess and check we see that $x = 1$
when we double check we see that $n(P) = 28$

- You can use the Principle of Inclusion and Exclusion to determine the number of elements in the union of three sets:

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$$

In our example:

$$28 = 13 + 13 + 13 - (x+3) - (x+2) - (x+4) + x$$

$$28 = 39 - x - 3 - x - 2 - x - 4 + x$$
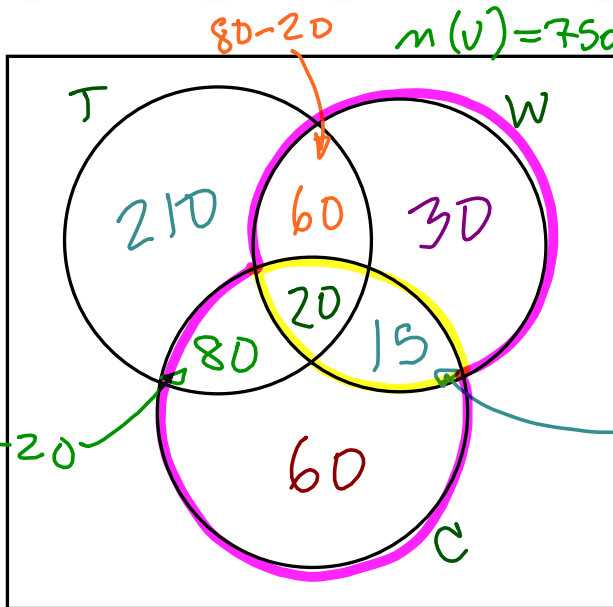
$$28 = 39 - 2x - 9$$

$$2x = 39 - 9 - 28$$

$$\frac{2x}{2} = \frac{2}{2} \implies \boxed{x = 1}$$

A

B

C

EXAMPLE 3 | Correcting errors that involve sets

Shannon's high school starts a campaign to encourage students to use "green" transportation for travelling to and from school. At the end of the first semester, Shannon's class surveys the 750 students in the school to see if the campaign is working. They obtain these results:

- 370 students use public transit. → $n(T) = 370$
- 100 students cycle and use public transit. → $n(C \cup T) = 100$
- 80 students walk and use public transit.
- 35 students walk and cycle.
- 20 students walk, cycle, and use public transit. → $n(C \cup T \cup W) = 20$
- 445 students cycle or use public transit.
- 265 students walk or cycle.

How many students use green transportation for travelling to and from school?

$80-20$ $n(U) = 750$

let $T = \{transit\}$
$W = \{walk\}$
$C = \{cycle\}$

Venn diagram with three overlapping circles labelled T, W, C. Regions: T only 210, W only 30, T∩W 60, centre 20, T∩C 80, W∩C 15, C only 60.

$100-20$

$35-20$

now $n(T) = 370$ so...

$T_{only} = 370 - 60 - 20 - 80$
$= 210$

walk or cycle $= 265$
but we've already
counted $60+20+80$
$+15+60 = 235$
∴ $265-235 = 30$

also c only $= 445 - 370 - 15$
$= 60$

Your Turn Total (sum of all) $= 475$

How many students use exactly one method of green transportation?

$60 + 210 + 30 = 300$