Homefun take up/issues/exultations...

9 a) $\frac{5x^3+x^2+3}{D(x)} = 5x^2-14x+42 - \frac{123}{D(x)}$

$5x^3+x^2+3 = D(x)(5x^2-14x+42)-123$

$(x+k)$

consider only the constants

$3 = 42k - 123$

$126 = 42k$

$\boxed{3 = k}$ $\therefore D(x) = x+3$

5 c) $\div$ by $2x+1 \Rightarrow 2x+1=0$

$2x=-1$

$x=-\frac{1}{2}$ k

$$\begin{array}{r|rrrr} -\frac{1}{2} & 2 & 5 & -4 & -5 \\ & & -1 & -2 & 3 \\ \hline & 2 & 4 & -6 & -2 \end{array}$$

answer $= 2x^2+4x-6 - \frac{2}{2x+1}$

The Remainder Theorem

Part 1 - When a polynomial P(x) is divided by x - a, the remainder is P(a).

$a = -4$

Ex 1: Find the remainder when $P(x) = 4x^3 - 2x^2 + 6x + 3$ is divided by $x + 4$

$$P(-4) = 4(-4)^3 - 2(-4)^2 + 6(-4) + 3$$
$$= -256 - 32 - 24 + 3$$
$$= -309$$

$a = -2$

Ex 2: When $2x^4 + kx^2 + 4x - 5$ is divided by $x + 2$ the remainder is 7. Determine the value of k.

$$P(-2) = 7$$
$$2(-2)^4 + k(-2)^2 + 4(-2) - 5 = 7$$
$$32 + 4k - 8 - 5 = 7$$
$$\frac{4k}{4} = \frac{-12}{4}$$
$$\boxed{k = -3}$$

$\to a = 2$

Ex 3: When $P(x) = 4x^3 + cx^2 + dx - 5$ is divided by $x - 2$ the remainder is 7.
When $P(x)$ is divided by $x + 1$ the remainder is also 7. What are the values of c and d?

$\to a = -1$

$$P(2) = 7$$
$$4(2)^3 + c(2)^2 + d(2) - 5 = 7$$
$$32 + 4c + 2d - 5 = 7$$
$$(4c + 2d = -20) \div 2$$
$$2c + d = -10 \quad ①$$

$$P(-1) = 7$$
$$4(-1)^3 + c(-1)^2 + d(-1) - 5 = 7$$
$$-4 + c - d - 5 = 7$$
$$c - d = 16 \quad ②$$

$$2c + d = -10 \quad ①$$
$$c - d = 16 \quad ②$$
$$3c = 6 \Rightarrow \boxed{\begin{array}{l} c = 2 \\ d = -14 \end{array}}$$

Ex 4: Find the remainder when $16x^3 - 2x^2 + x - 4$ is divided by $2x - 3$

$$2x - 3 = 0$$
$$2x = 3$$
$$x = 3/2$$

$a = 3/2$

$$P(3/2) = 16\left(\tfrac{3}{2}\right)^3 - 2\left(\tfrac{3}{2}\right)^2 + \left(\tfrac{3}{2}\right) - 4$$
$$= 47$$

Remainder theorem Part II

When a polynomial P(x) is divided by ax - b the remainder is $P\left(\frac{b}{a}\right)$

The Factor Theorem

Find the remainder when $P(x) = x^3 - 2x^2 - 5x + 6$ is divided by...

(a) $x - 2$

$P(2)$
$= 2^3 - 2(2)^2 - 5(2) + 6$
$= 8 - 8 - 10 + 6$
$= -4$

(b) $x - 1$

$P(1)$
$= (1)^3 - 2(1)^2 - 5(1) + 6$
$= 1 - 2 - 5 + 6$
$= 0$

(c) $x + 5$

$P(-5)$
$= -125 - 50 + 25 + 6$
$= -144$

$x - 1$ must be a factor of $P(x)$ since there is no remainder!

Anything special? ... oh and $(x-2)$ and $(x+5)$ are NOT

How about x + 2? or x-3? or x+7?

$P(-2)$
$= -8 - 8 + 10 + 6$
$= 0$

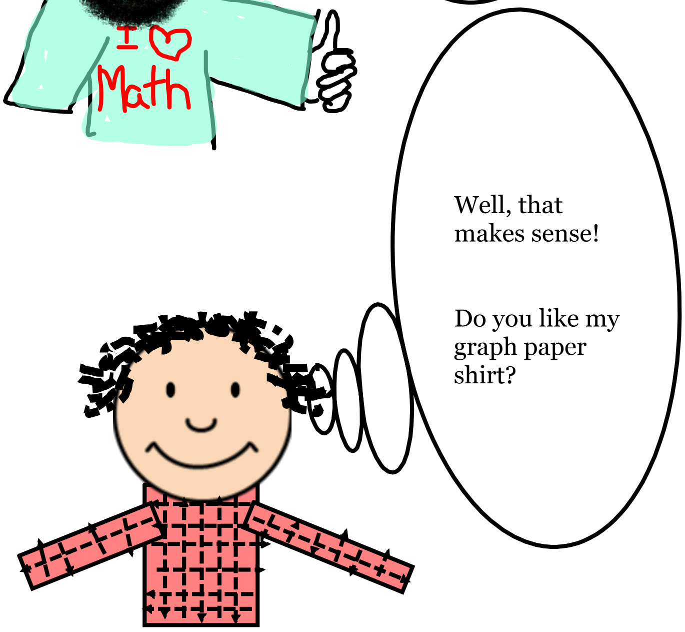
both factors!!

$P(3)$
$= 0$

$P(-7)$
$= 216$

The factor theorem: $x - a$ is a factor of $P(x)$ if and only if $P(a) = 0$

Example 3: Factor **$P(x) = x^5 - 2x^4 - 13x^3 + 26x^2 + 36x - 72$**



For polynomials of degree greater than 3, you may need to do the factor theorem more than once.

Factor Theorem - find a factor of $P(x)$, then divide to find the other factors.

Example 1: Factor $4x^4 + 6x^3 - 6x^2 - 4x$

$P(x) = x(4x^3 + 6x^2 - 6x - 4)$

I could take a guess at a factor?!

or I could consider factors of -4

start with $\pm1, \pm2, \pm4$

test $P(-2) = 0 \Rightarrow x+2$ is a factor so divide $P(x)$ by $x+2$

$$\begin{array}{r|rrrr} -2 & 4 & 6 & -6 & -4 \\ & & -8 & 4 & 4 \\ \hline & 4 & -2 & -2 & 0 \end{array}$$

now..

$P(x) = x(x+2)(4x^2 - 2x - 2)$

$= 2x(x+2)(2x^2 - x - 1)$

$= 2x(x+2)(2x+1)(x-1)$

Example 2: Sketch the graph of $f(x) = x^4 - 6x^3 + 2x^2 - 12x$

$= x(x^3 - 6x^2 + 2x - 12)$

check factors $\pm1, \pm2, \pm3, \pm4, \pm6, \pm12$

test $(x-1)$:

$P(1) = (1)^3 - 6(1)^2 + 2(1) - 12 \neq 0$

test $(x+1)$

$P(-1) = (-1)^3 - 6(-1)^2 + 2(-1) - 12$

$= -1 - 6 - 2 - 12 \neq 0$

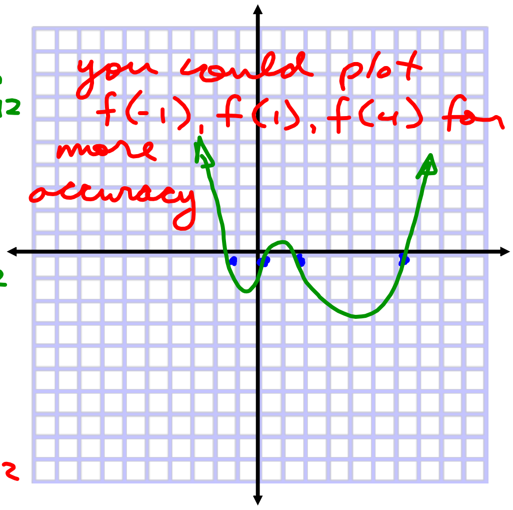
test $(x-2)$

$P(2) = (2)^3 - 6(-2)^2 + 2(-2) - 12$

$= 8 - 24 - 4 - 12 \neq 0$

$(x+2)$ $P(-2) = (-2)^3 - 6(-2)^2 + 2(-2) - 12$

$= -8 - 24 - 4 - 12 \neq 0$



test $(x-6)$

$P(6) = (6)^3 - 6(6)^2 + 2(6) - 12 = 0$ $\therefore (x-6)$ is a factor

Now use synthetic division

$$\begin{array}{r|rrrr} 6 & 1 & -6 & 2 & -12 \\ & & 6 & 0 & 12 \\ \hline & 1 & 0 & 2 & 0 \end{array}$$

$R(x) = 0$

Since $x-6$ is a factor

quotient is $x^2 - 2$

$x^2 - 2 = 0$

$x^2 = 2$

$x = \pm\sqrt{2}$

$(x+\sqrt{2})(x-\sqrt{2})$

Since we can factor quadratics easily (or use quadratic formula) the goal is to divide until we reach a quadratic polynomial.

$P(x) = x(x-6)(x^2-2)$

$= x(x-6)(x+\sqrt{2})(\sqrt{x}-\sqrt{2})$

Example 4: When 2x3 - mx2 + nx -2 is divided by x + 1, the remainder is -12.
Also, x - 2 is a factor. Determine values of m and n.

$P(-1) = -12$

$-12 = 2(-1)^3 - m(-1)^2 + n(-1) - 2$

$-12 = -2 - m - n - 2$

$m + n = 8$ ①

$P(2) = 0$

$2(2)^3 - m(2)^2 + n(2) - 2 = 0$

$16 - 4m + 2n - 2 = 0$

$(-4m + 2n = -14) \div (-2)$

$2m - n = 7$ ②

① + ②

$m + n = 8$

$+\ 2m - n = 7$

$3m = 15$

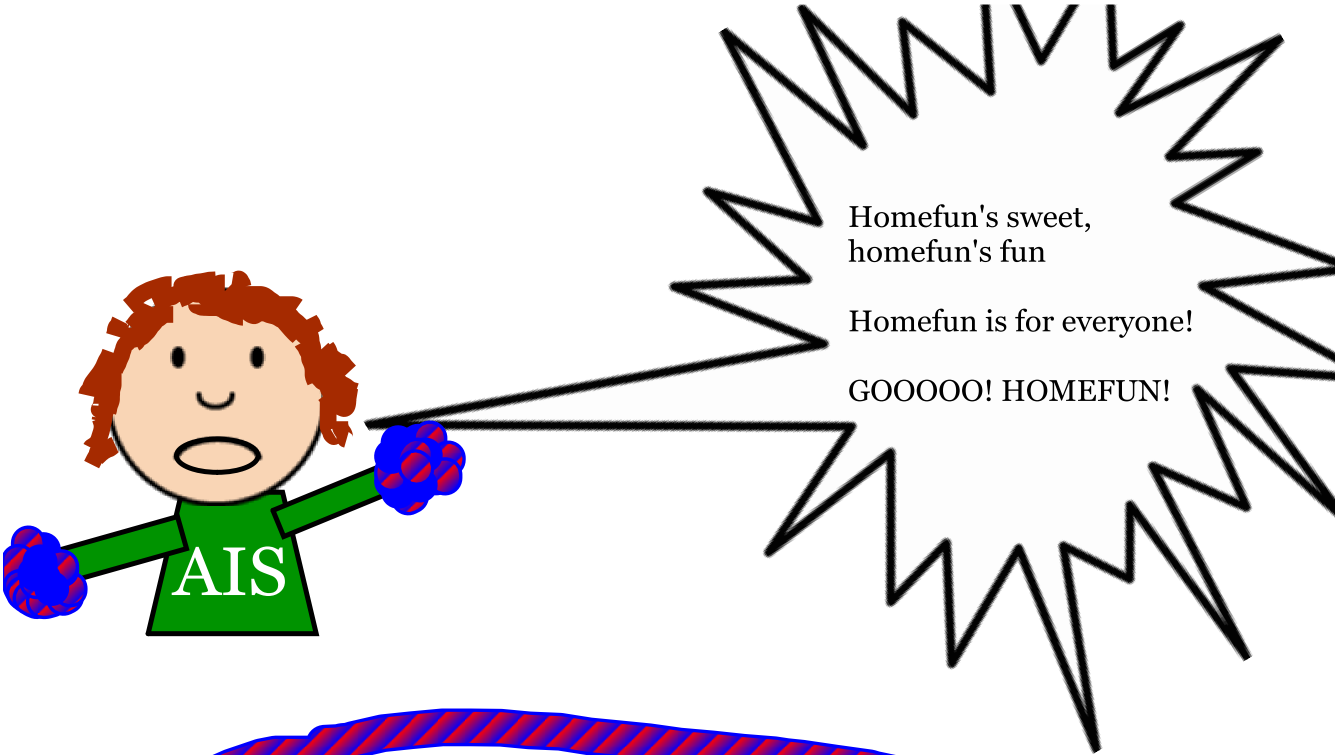
$\boxed{m = 5}$

sub $m = 5$ into ①

$5 + n = 8$

$\boxed{n = 3}$

$\therefore f(x) = 2x^3 - 5x^2 + 3x - 2$



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