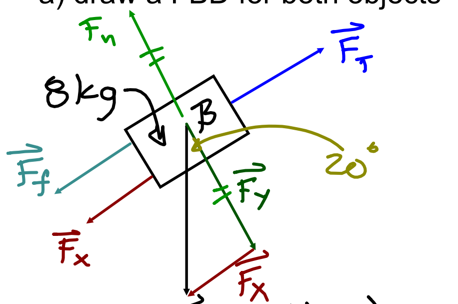
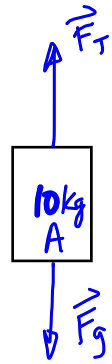
3.8 Pulleys, Planes, Springs and Friction

ex. An 8.0 kg box rests on an inclined plane (20.0°) with a coefficient of kinetic friction of 0.25. It is tethered to a 10 kg mass as is depicted in the diagram below.

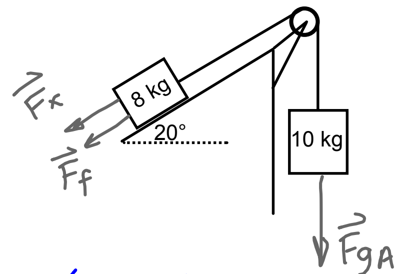
a) draw a FBD for both objects



$\vec{F}_{g_B} = (8)(9.8) = 78.4\text{ N}$



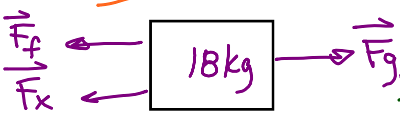
$\vec{F}_{g_A} = (10)(9.8)$
$= 98\text{ N}$



b) calculate the acceleration of the system

consider the total mass of 18 kg... it all accelerates the same.

total: $\vec{F}_{net} = \vec{F}_{g_A} - \vec{F}_f - \vec{F}_x$



$\vec{F}_{g_A} = 98\text{ N}$

$\vec{F}_x = F_{g_B}\sin 20°$
$= (8)(9.8)\sin 20°$
$= 26.81\text{ N}$

$\vec{F}_n = \vec{F}_y = F_{g_B}\cos 20°$
$= (8)(9.8)\cos 20°$
$= 73.67\text{ N}$

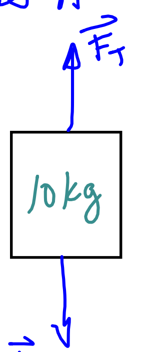
$\therefore \vec{F}_f = \mu F_n$
$= (0.25)(73.67)$
$= 18.42\text{ N}$

$\vec{F}_{net} = 98\text{ N} - 18.42$
$- 26.81$
$\boxed{\vec{F}_{net} = 52.77\text{ N}}$

Now $\vec{a} = \dfrac{\vec{F}_{net}}{m_{total}}$
$= \dfrac{52.77\text{ N}}{18\text{ kg}}$
$= 2.93\text{ m/s}^2$ [up the ramp]

c) calculate the tension in the string

consider the simpler FBD... mass A



$\vec{F}_{g_A} = 98\text{ N}$

$\vec{F}_{net} \neq 0$
since $\vec{a} = 2.93\text{ m/s}^2$
$m = 10\text{ kg}$

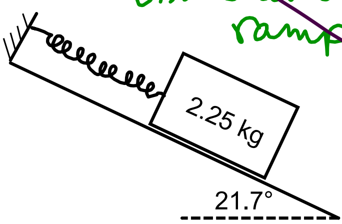
$\therefore \vec{F}_{net_A} = (10)(2.93)$
$= 29.3\text{ N}$ [down]

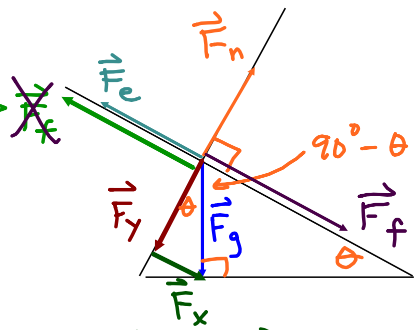
$\vec{F}_{net_A} = \vec{F}_{gA} - \vec{F}_T$

$\vec{F}_T = \vec{F}_{gA} - \vec{F}_{net_A}$
$= 98\text{ N} - 29.3\text{ N}$
$\boxed{\vec{F}_T = 68.7\text{ N}}$ [up]

ex. Consider the arrangement below: the spring is currently stretched $\boxed{23.5}$ cm from its rest position and has a spring constant of 125 N/m, the mass is 2.25 kg, the pulley is frictionless but the ramp/box have a coefficient of kinetic friction of 0.222, the ramp is inclined at 21.7°. What is the initial acceleration of the box?

~~assume box slides down ramp~~





$\vec{F}_e = k\Delta x$
$= (125\ N/m)(.235\ m)$
$= 29.375\ N\ [up]$

$\vec{F}_x = \vec{F}_g \sin 21.7°$
$= (2.25)(9.8)\sin 21.7°$
$= 8.153\ N\ [down]$

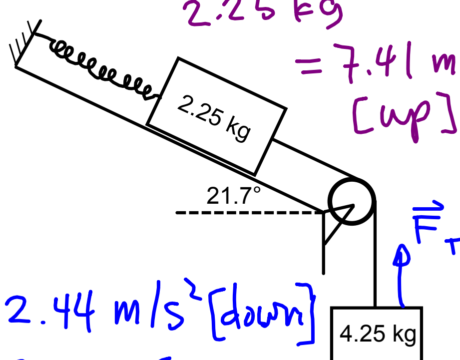
Since $\vec{F}_e > \vec{F}_x$, the box actually moves UP the ramp, so we should change the direction of $\vec{F}_f$

now $\vec{F}_n = \vec{F}_y = \vec{F}_g \cos\theta$
$= (2.25)(9.8)\cos 21.7°$
$= 20.49\ N$

and $\vec{F}_f = \mu \vec{F}_n$
$= (0.222)(20.49\ N)$
$= 4.55\ N$

$\vec{F}_{net_x} = \vec{F}_e - \vec{F}_f - \vec{F}_x$
$= 29.375 - 4.55 - 8.153$
$= 16.672\ N$ [up ramp]

$\therefore \vec{a} = \dfrac{\vec{F}_{net_x}}{m}$
$= \dfrac{16.672\ N}{2.25\ kg}$
$= 7.41\ m/s^2$ [up]



Homework: The system above now has a 4.25 kg mass attached over a pulley as seen in the diagram to the right. Find the initial acceleration now.

answer: $\vec{a} = 2.44\ m/s^2$ [down]
$\vec{F}_T = 31.3\ N$ [up]