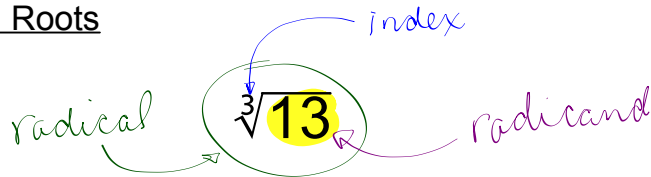


### 4.1 Estimating Roots

#### Terminology



#### Squares and Cubes

base	1	2	3	4	5	6	7	8	9	10	11	12	13
square	1	4	9	16	25	36	49	64	81	100	121	144	169
cube	1	8	27	64	125	216	343	512	729	1000	1331	1728	2197

\* To effectively estimate the square root of a number we must know the **perfect** squares above and below the given number.

a)  $\sqrt{30}$

$$\sqrt{25} < \sqrt{30} < \sqrt{36}$$

$$5 < \sqrt{30} < 6$$

$\sqrt{30} \approx 5.5$   
 since 30 is about halfway between 25 and 36

b)  $\sqrt{52}$

$$\sqrt{49} < \sqrt{52} < \sqrt{64}$$

$$7 < \sqrt{52} < 8$$

$\therefore \sqrt{52} \approx 7.2$

c)  $\sqrt{312}$

$$\sqrt{289} < \sqrt{312} < \sqrt{324}$$

$$17 < \sqrt{312} < 18$$

$\therefore \sqrt{312} \approx 17.7$

\* For cube roots, we must know the **perfect** cubes above and below the given number.

d)  $\sqrt[3]{20}$

$$\sqrt[3]{8} < \sqrt[3]{20} < \sqrt[3]{27}$$

$$2 < \sqrt[3]{20} < 3$$

$\sqrt[3]{20} \approx 2.7$

e)  $\sqrt[3]{600}$

$$\sqrt[3]{512} < \sqrt[3]{600} < \sqrt[3]{729}$$

$$8 < \sqrt[3]{600} < 9$$

$\therefore \sqrt[3]{600} \approx 8.4$

f)  $\sqrt[3]{-29}$

$$\sqrt[3]{-64} < \sqrt[3]{-29} < \sqrt[3]{-27}$$

$$-4 < \sqrt[3]{-29} < -3$$

$\therefore \sqrt[3]{-29} \approx -3.1$

How about these?

a)  $\sqrt{-9} = \text{error}$

... not possible since no 2 identical #'s have a negative product  
 $(-)(-) = (+)$

b)  $\sqrt[4]{-625}$

also NOT possible  
 $(-)(-)(-)(-) = (+)$

c)  $\sqrt[3]{-27} = -3$

since  $(-3)^3 = -27$

calculator tip: exponent  $\square \wedge$

or  $\square x^y$  or  $\square y^x$

root  $\square \sqrt[x]{y}$  or  $\square \sqrt{y^x}$

on a Ti-83/4

$\square \text{Math} \square \square$

try  $\sqrt[5]{65}$   
 $\hat{=} 2.30$

\* It is impossible to take an **even** root of a negative number. However, **odd** roots are possible.

a)  $\sqrt{-9} = \emptyset$

b)  $\sqrt[3]{-27} = -3$

c)  $\sqrt[4]{-625} = \text{NOT possible}$

\* A root is an exact value if its value can be written as a **rational number** (fraction). A number that **repeats** or **terminates** can always be written as a **fraction**.

a)  $\sqrt{1.21}$

= 1.1

=  $\frac{11}{10}$

Fraction

exact value... if terminates

b)  $\sqrt{\frac{16}{81}}$

= 0.4

repeats so exact value

=  $\frac{4}{9}$

c)  $\sqrt{\frac{2}{3}}$

= 0.816496...

- no pattern

- does not terminate

∴ not rational

∴ approximate value only

ex/a)  $0.\overline{2} = \frac{2}{9}$

b)  $0.\overline{35} = \frac{35}{99}$

c)  $0.\overline{123} = \frac{123}{999}$

d)  $0.1\overline{23} = \frac{123}{1000}$

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