4.1 Estimating Roots

Terminology

radical $\sqrt[3]{13}$ index radicand

Squares and Cubes

base	1	2	3	4	5	6	7	8	9	10	11	12	13
square	1	4	9	16	25	36	49	64	81	100	121	144	169
cube	1	8	27	64	125	216	343	512	729	1000	1331	1728	2197

* To effectively estimate the square root of a number we must know the perfect squares above and below the given number.

a) $\sqrt{30}$

$\sqrt{25} < \sqrt{30} < \sqrt{36}$

$\underbrace{\sqrt{25} < \sqrt{30}}_{5} \underbrace{< \sqrt{36}}_{6}$

$5 < \sqrt{30} < 6$

$\sqrt{30} \approx 5.5$

Since 30 is about halfway between 25 and 36

b) $\sqrt{52}$

$\underbrace{\sqrt{49} < \sqrt{52}}_{3} \underbrace{< \sqrt{64}}_{12}$

$7 < \sqrt{52} < 8$

$\therefore \sqrt{52} \approx 7.2$

c) $\sqrt{312}$

$\underbrace{\sqrt{289} < \sqrt{312}}_{23} \underbrace{< \sqrt{324}}_{12}$

$17 < \sqrt{312} < 18$

$\therefore \sqrt{312} \approx 17.7$

* For cube roots, we must know the perfect cubes above and below the given number.

d) $\sqrt[3]{20}$

$\underbrace{\sqrt[3]{8} < \sqrt[3]{20}}_{12} \underbrace{< \sqrt[3]{27}}_{7}$

$2 < \sqrt[3]{20} < 3$

$\sqrt[3]{20} \approx 2.7$

e) $\sqrt[3]{600}$

$\underbrace{\sqrt[3]{512} < \sqrt[3]{600}}_{88} \underbrace{< \sqrt[3]{729}}_{129}$

$8 < \sqrt[3]{600} < 9$

$\therefore \sqrt[3]{600} \approx 8.4$

f) $\sqrt[3]{-29}$

$\underbrace{\sqrt[3]{-64} < \sqrt[3]{-29}}_{35} \underbrace{< \sqrt[3]{-27}}_{2}$

$-4 < \sqrt[3]{-29} < -3$

$\therefore \sqrt[3]{-29} \approx -3.1$

How about these?

a) $\sqrt{-9}$ = error

... not possible since no 2 identical #s have a negative product

$(-)(-) = (+)$

b) $\sqrt[4]{-625}$ also NOT possible

$(-)(-)(-)(-) = (+)$

c) $\sqrt[3]{-27} = -3$

Since $(-3)^3 = -27$

calculator tip: exponent $\boxed{\wedge}$ or $\boxed{x^y}$ or $\boxed{y^x}$

root $\boxed{\sqrt[x]{y}}$ or $\boxed{\sqrt[y]{x}}$

on a Ti-83/4

$\boxed{\text{Math}}$ $\boxed{5}$

try $\sqrt[5]{65}$

$\doteq 2.30$

* It is impossible to take an even root of a negative number. However, odd roots are possible.

a) $\sqrt{-9} = \emptyset$

b) $\sqrt[3]{-27} = -3$

c) $\sqrt[4]{-625}$ = Not possible

* A root is an exact value if its value can be written as a rational number (fraction). A number that repeats or terminates can always be written as a fraction.

a) $\sqrt{1.21}$

$= 1.1$

$= \frac{11}{10}$ Fraction

exact value... it terminates

b) $\sqrt{\frac{16}{81}}$

$= 0.\overline{4}$

repeats so exact value

$= \frac{4}{9}$

c) $\sqrt{\frac{2}{3}}$

$= 0.816496...$

- no pattern
- does not terminate

∴ not rational

∴ approximate value only

ex/a) $0.\overline{2} = \frac{2}{9}$ b) $0.\overline{35} = \frac{35}{99}$

c) $0.\overline{123} = \frac{123}{999}$ d) $0.123 = \frac{123}{1000}$

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