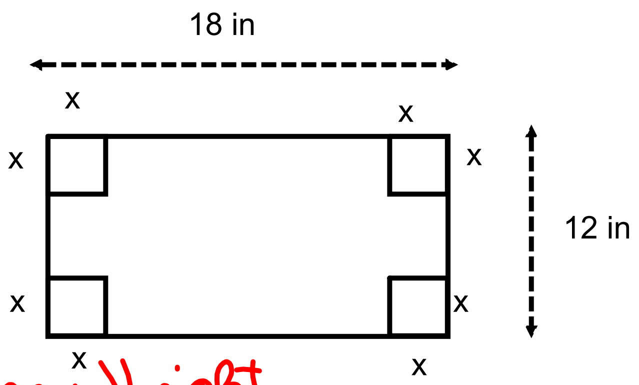
In your group create the box with the largest volume.



Recall:

V = (base area) height

4.1 Solving Polynomial Equations

Recall: Finding zeroes of a polynomial function.

Example: Find the zeroes of $f(x) = x^3 - 5x^2 - 2x + 24$

test factors of ± 24 : $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6 \ldots$

$$\left.\begin{aligned} f(-2) &= (-2)^3 - 5(-2)^2 - 2(-2) + 24 \\ &= -8 - 20 + 4 + 24 = 0 \end{aligned}\right\} \therefore \boxed{(x+2)}$$

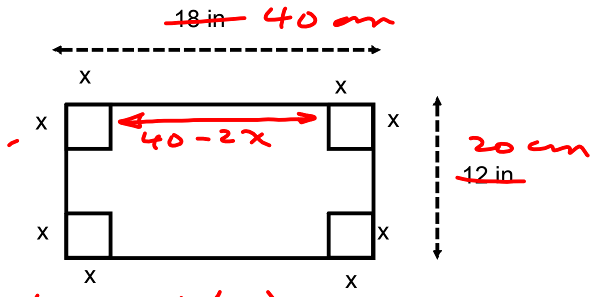
$$\begin{array}{r|rrrr} -2 & 1 & -5 & -2 & 24 \\ & & -2 & 14 & -24 \\ \hline & 1 & -7 & 12 & 0 \end{array} \Rightarrow \begin{aligned} &x^2 - 7x + 12 \\ &= (x-3)(x-4) \end{aligned}$$

$$\therefore f(x) = (x+2)(x-3)(x-4)$$

This is the same as....

Thus when we solve ...

Example 1: Solve for the dimensions of the box created from the piece of paper below that give a volume of 1500cm³



$V = (L)(W)(h)$

$1500 = (40-2x)(20-2x)(x)$

I need one side to = 0

$0 = (800 - 120x + 4x^2)(x) - 1500$

$0 = 4x^3 - 120x^2 + 800x - 1500$

$0 = 4(x^3 - 30x^2 + 200x - 375)$

test $P(5) = 0$ yay! ∴ $(x-5)$ is a factor

$$\begin{array}{r|rrrr} 5 & 1 & -30 & 200 & -375 \\ & & 5 & -125 & 375 \\ \hline & 1 & -25 & 75 & 0 \end{array} \Rightarrow x^2 - 25x + 75$$

use quadratic formula.

$$x = \frac{-(-25) \pm \sqrt{(-25)^2 - 4(1)(75)}}{2(1)}$$

$$= \frac{25 \pm \sqrt{325}}{2}$$

$$= \frac{25 \pm 18.02}{2}$$

$x = 21.5$ and $x \doteq 3.5$

∴ The volume eqnⁿ has roots @ $x = 5$, $x = 3.5$ and $x = 21.5$ cm

To have $V = 1500\ cm^3$, we can cut squares of 5 cm or 3.5 cm in length

consider domain

$\{x \in \mathbb{R} \mid 0 < x < 10\}$

Example 2: Solve $4x^3 - 12x^2 - x = -3$ $\Rightarrow 4x^3 - 12x^2 - x + 3 = 0$

test $x = \frac{\pm 1}{1}, \frac{\pm 3}{1}, \underbrace{\frac{\pm 1}{4}, \pm\frac{3}{4}, \frac{\pm 1}{2}, \frac{\pm 3}{2}}$

If you have a leading coefficient $\neq 1$, you must consider all factors of $\frac{p}{q}$

constant

leading coefficient

calc says $P(3) = 0$

$$\begin{array}{r|rrrr} 3 & 4 & -12 & -1 & 3 \\ & & 12 & 0 & 3 \\ \hline & 4 & 0 & -1 & 0 \end{array} \Rightarrow 4x^2 - 1 = 0$$

$$(2x+1)(2x-1) = 0$$

$\therefore P(x) = (x-3)(2x+1)(2x-1)$

roots @ $x = 3, \pm\frac{1}{2}$

Unsolvable/Unfactorable Polynomials

(a) $\underbrace{x^4 + 5x^2}_{f(x)} = \underbrace{-1}_{g(x)}$

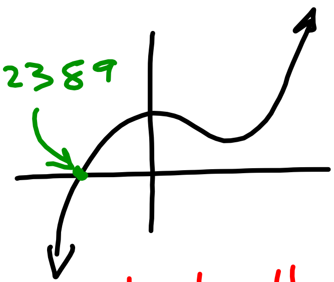
where do $f(x)$ and $g(x)$ intersect?

NEVER

x^4 and $5x^2$ can never be negative

(b) $x^3 - 2x + 3 = 0$

$x = -1.892389$



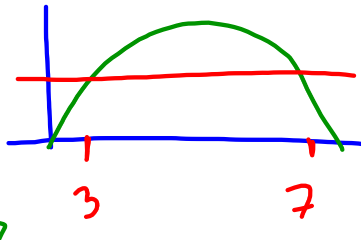
not at all factorable...

and the other root is imaginary

Example 3: A boy launches a paper airplane and its height is modelled by $h(t) = -5t^2 + 50t$. At the same time the boy's penguin friend launches his paper airplane, outfitted with a Quantum Ninja Extreme Rocket Engine. The penguin's "jet" has its height modelled by $h(t) = 2t^3 - 21t^2 + 74t$

(a) At what time(s) does the boy's plane reach a height of 105m?

$$105 = -5t^2 + 50t$$
$$0 = -5t^2 + 50t - 105$$
$$0 = -5(t^2 - 10t + 21)$$
$$0 = -5(t-3)(t-7)$$
$$\therefore t = 3 \text{ and } t = 7$$



(b) At what time(s) does the penguin's jet reach a height of 175m?

$$175 = 2t^3 - 21t^2 + 74t$$
$$0 = 2t^3 - 21t^2 + 74t - 175$$
$$P(7) = 0 \quad \therefore (t-7) \text{ is a factor}$$

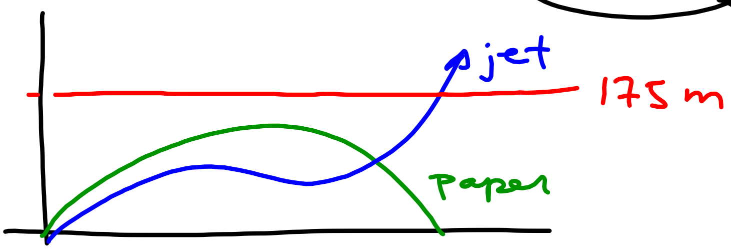
$$\begin{array}{r|rrrr} 7 & 2 & -21 & 74 & -175 \\ & & 14 & -49 & 175 \\ \hline & 2 & -7 & 25 & 0 \end{array}$$

$2t^2 - 7t + 25$ in quad. form.

$b^2 - 4ac < 0$

no real roots

$$P(t) = (t-7)(2t^2 - 7t + 25)$$



(c) At what time(s) are the two aircraft at the same height?

set 2 equ^ns equal to one another

$$-5t^2 + 50t = 2t^3 - 21t^2 + 74t$$
$$0 = 2t^3 - 16t^2 + 24t$$
$$0 = 2t(t^2 - 8t + 12)$$
$$0 = 2t(t-2)(t-6)$$

$\therefore$ the aircraft have the same height @ $t = 0, 2, 6$ seconds.

Homefun Wow

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