4.2 Irrational Numbers

* A rational number is a number that can be written as a fraction. When expressed as a decimal, it either terminates or repeats.

ex. $\frac{2}{3} = 0.66666\ldots = 0.\overline{6}$ repeats

ex. $\sqrt[3]{1.331} = 1.1$ terminates

$\therefore$ both rational

* All integers are rational numbers

ex. $3 = \frac{3}{1} = \frac{6}{2} = \frac{9}{3} = \frac{300}{100}$

ex. Rational or irrational?

a) $\sqrt{\frac{25}{81}} = 0.\overline{5}$ rational

b) $\sqrt[3]{26} = 2.962496\ldots$ irrational

c) $\sqrt{\frac{9}{100}} = \frac{3}{10} = 0.3$ rational

* The set of REAL Numbers: $\mathbb{R}$

Rational ($\mathbb{Q}$): $\sqrt{\frac{9}{100}}$, $3/4$, $-\frac{4}{9}$, -17

Integers ($\mathbb{Z}$): $\ldots -3, -2, -1, 0, 1, 2, \ldots$

Whole numbers (W): $0, 1, 2, 3, \ldots$

Natural Numbers: ($\mathbb{N}$) $1, 2, 3, 4, \ldots$

Irrational Numbers: $\sqrt{2}$, π, $-\sqrt[3]{7}$

Imaginary

$\sqrt{-1} = i$ even index

$i^2 = (\sqrt{-1})^2 = -1$

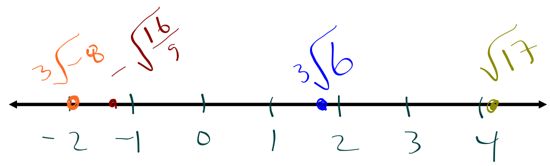
ex. Place the following numbers on a number line

$\sqrt{17}$ ≈ 4.15

$\sqrt[3]{-8}$ $= -2$

$\sqrt[3]{6}$ ≈ 1.8

$-\sqrt{\frac{16}{9}}$ $= -\frac{4}{3} = -1.\overline{3}$



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4.3 Mixed and Entire Radicals

Property: we can decompose a radical with multiplication or division but NOT with addition or subtraction.

$$\boxed{\sqrt[n]{a\cdot b} = \sqrt[n]{a}\cdot\sqrt[n]{b}} \quad \text{and} \quad \boxed{\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}}$$

⚠ indices must be the same

Beware!

$\sqrt[n]{a+b} \neq \sqrt[n]{a} + \sqrt[n]{b}$ Also $\sqrt[n]{a-b} \neq \sqrt[n]{a} - \sqrt[n]{b}$

ex. $\sqrt{99} = \sqrt{9\times 11} = \sqrt{9}\times\sqrt{11} = 3\times\sqrt{11} = 3\sqrt{11}$ mixed radical

entire form

* A radical is in simplified form when we have removed all the perfect square factors from the radicand in the case of a square root (perfect cube factors in the case of a cube root).

* A radical is in entire form when it has no coefficient ;everything is under the root sign.

ex. Simplify

a) $\sqrt{72}$
$= \sqrt{9\times 8}$
$= \sqrt{9}\times\sqrt{8}$
$= 3\sqrt{8}$
not fully simplified
$= 3\sqrt{4\times 2}$
$= 3\times\sqrt{4}\times\sqrt{2}$
$= 3\times 2\times\sqrt{2}$
$= 6\sqrt{2}$

b) $\sqrt[3]{72}$
$= \sqrt[3]{8\times 9}$
$= \sqrt[3]{8}\times\sqrt[3]{9}$
$= 2\sqrt[3]{9}$

c) $\sqrt[4]{128}$
$= \sqrt[4]{16\times 8}$
$= \sqrt[4]{16}\times\sqrt[4]{8}$
$= 2\sqrt[4]{8}$

ex. Change to entire form

a) $2\sqrt{7}$
$= 2\times\sqrt{7}$
$= \sqrt{4}\times\sqrt{7}$
$= \sqrt{4\times 7} = \sqrt{28}$

b) $2\sqrt[3]{4}$
$= \sqrt[3]{8}\times\sqrt[3]{4}$
$= \sqrt[3]{8\times 4}$
$= \sqrt[3]{32}$

c) $3\sqrt[4]{2}$
$= \sqrt[4]{81}\times\sqrt[4]{2}$
$= \sqrt[4]{81\times 2} = \sqrt[4]{162}$

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ex// $\sqrt{98}$
$= \sqrt{49\times 2}$
$= \sqrt{49}\times\sqrt{2}$
$= 7\sqrt{2}$

ex// $3\sqrt[3]{2}$
$= \sqrt[3]{27}\times\sqrt[3]{2}$
$= \sqrt[3]{54}$