4.2 Permutations and Factorial Notation

permutations: an arrangement of distinguishable elements in a different order.

ex. how many permutations exist for the lettres A, B and C

ABC BAC CAB
ACB BCA CBA $\}$ $\underline{3} \cdot \underline{2} \cdot \underline{1} = 6$

factorial notation: the product of consecutive descending natural numbers

1, 2, 3, . . .

ex. 5! =

$5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

$= 120$

$\boxed{5}\ \boxed{\text{math}}\ \boxed{\text{PRB}}\ \boxed{!}$ 4:

but also... 3! x 4 =

$= (3 \cdot 2 \cdot 1) \times 4$

$= 1 \cdot 2 \cdot 3 \cdot 4$

$= 4!$

EXAMPLE 1 Solving a counting problem where order matters

Determine the number of arrangements that six children can form while lining up to drink.

$\underline{6} \cdot \underline{5} \cdot \underline{4} \cdot \underline{3} \cdot \underline{2} \cdot \underline{1} = 6!$

$= 720$

EXAMPLE 2 Evaluating numerical expressions involving factorial notation

Evaluate the following.

a) 10!

$= 10 \cdot 9 \cdot 8 \cdot \ldots$

$= 3\,628\,000$

b) $\frac{12!}{9!3!}$ $= \frac{12 \cdot 11 \cdot 10 \cdot \cancel{9!}}{3!\ \cancel{9!}}$

$= \frac{\cancel{12}^{4} \cdot 11 \cdot \cancel{10}^{5}}{\cancel{3}^{1} \cdot \cancel{2}^{1} \cdot 1}$

$= \frac{4 \cdot 11 \cdot 5}{1} = 20 \cdot 11 = \boxed{220}$

$\frac{\smile}{\frown} = 1$

Your Turn

When $(n + 3)(n + 2)!$ is multiplied by $(n + 4)$ and then divided by $(n + 2)!$, what is the result?

$$\frac{(n+4)(n+3)\cancel{(n+2)!}}{\cancel{(n+2)!}} = (n+4)(n+3)$$

EXAMPLE 4 Solving an equation involving factorial notation

get the value of n that satisfies the eqn

Solve $\frac{n!}{(n-2)!} = 90$, where $n \in I$.

$$\frac{n\cdot(n-1)\cdot\cancel{(n-2)!}}{\cancel{(n-2)!}} = 90$$

* always expand the greater factorial so that it matches the lower factorial
* then cancel the terms with factorials
* solve for n

$n(n-1) = 90$

here we need 2 consecutive numbers that have a product = 90

* by observation $10 \cdot 9 = 90$

n is the bigger # so $\boxed{n=10}$

or factor

$n^2 - n - 90 = 0$ $\quad -10 \cdot 9 = 90$ $\quad -10 + 9 = -1$

$(n-10)(n+9) = 0$

$\boxed{n=10}$ $\quad \boxed{\cancel{n=-9}}$

Your Turn

Solve $\frac{(n+4)!}{(n+2)!} = 6$, where $n \in I$.

$$\frac{(n+4)(n+3)\cancel{(n+2)!}}{\cancel{(n+2)!}} = 6$$

$(n+4)(n+3) = 6$

$3 \cdot 2 = 6$

$\therefore n+4 = 3$ $\quad -4$

$\boxed{n=-1}$

OR

$n^2 + 7n + 12 = 6$

$n^2 + 7n + 6 = 0$

$(n+1)(n+6) = 0$

$n+1=0$ $\quad n+6=0$

$\boxed{n=-1}$ $\quad \boxed{\cancel{n=-6}}$

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check $(n+2)!$

$(-6+2)!$

not possible $\rightarrow (-4)!$

so $n \neq -6$

$\boxed{n=-1}$