4.2 Solving Linear Inequalities

Recall: Solving a linear equation... Solve $4x - 9 = 27$

$$\begin{aligned} 4x - 9 &= 27 \\ +9 \quad & \quad +9 \end{aligned}$$

$$\frac{4x}{4} = \frac{36}{4}$$

$$\boxed{x = 9}$$

Operations with inequalities: Consider $4 < 12$

add to both sides

$4 < 12$
$+2 \quad +2$
$6 < 14$ true

subtract from both sides

$4 < 12$
$-2 \quad -2$
$2 < 10$
still true

mult by +

$4(2) < 12(2)$
$8 < 24$
true

mult by -

$4(-2) < 12(-2)$
$-8 < -24$
False

divide by +

$\frac{4}{2} < \frac{12}{2}$
$2 < 6$
true

divide by -

$\frac{4}{-2} < \frac{12}{-2}$
$-2 < -6$
false

In these cases we must switch the direction of the inequality to make it true.

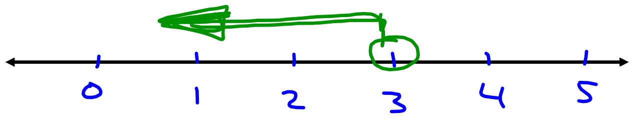
this is the only difference b/w solving an inequality and an eqn

Solve a linear inequality: Remember if we multiply or divide by a negative, the inequality sign switches.

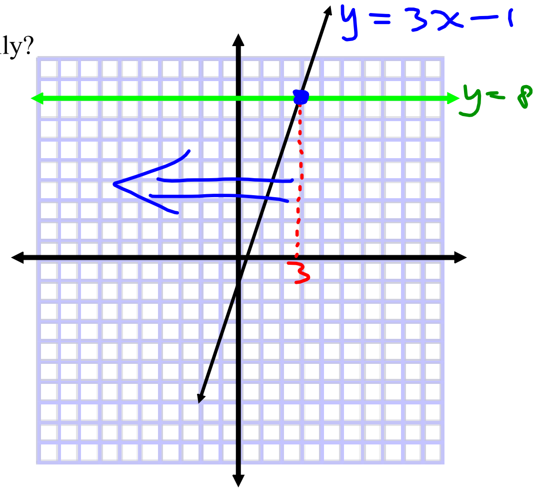
Solve: $3x - 1 < 8$ $+1$ $+1$

$\frac{3x}{3} < \frac{9}{3}$

$x < 3$



What does this mean graphically?



More examples:

(a) $35 - 2x \geq 20$

$-2x \geq 20 - 35$

$\frac{-2x}{-2} \geq \frac{-15}{-2}$

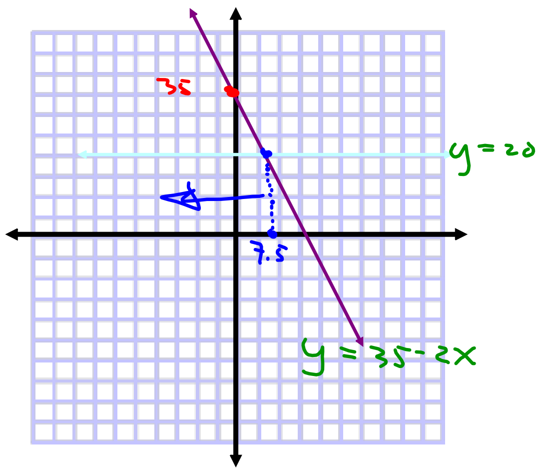
$x \leq 7.5$

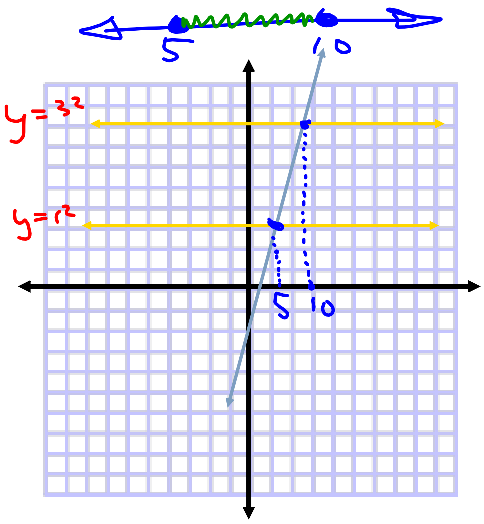
(b) $12 \leq 4x - 8 \leq 32$ $+8$
$+8$ $+8$

$\frac{20}{4} \leq \frac{4x}{4} \leq \frac{40}{4}$

$5 \leq x \leq 10$

What do these mean graphically?





Don't forget we can also use interval notation to discuss the solution sets!

$x \in (-\infty, 7.5]$

$x \in [5, 10]$

Example: Is x = 5 in the solution set of the following inequality?

$$4x-4\leq 9+2(x-4)$$

test $x = 5$

$LS = 4(5) - 4$
$= 20 - 4$
$= 16$

$RS = 9 + 2(5-4)$
$= 9 + 2(1)$
$= 11$

is $LS \leq RS$?
$16 \leq 11$ False

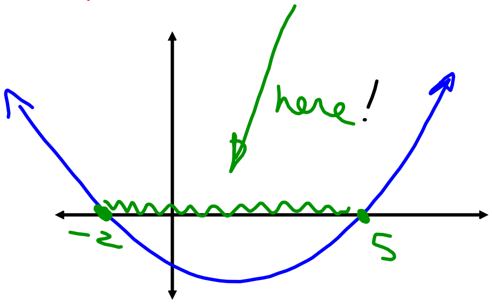
$\therefore$ $x = 5$ is not in the sol^n interval!

Example: Solve $x^2 - 3x - 10 < 0$

graphical approach

$(x-5)(x+2) < 0$

where is the function less than zero?



$\therefore -2 < x < 5$

or $x \in (-2, 5)$

test pts in the inequality

the roots are $x = -2, 5$ so test points around the roots... start with $x = 0$, since it is easy!

$(0)^2 - 3(0) - 10 < 0$

$-10 < 0$ ✓true

$\therefore$ the sol^n interval contains $x = 0$... using our knowledge of polynomials (continuity), conclude that if $x = 0$ satisfies the sol^n then $x \in (-2, 5)$ also satisfies the sol^n

or $-2 < x < 5$

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