4.3 Permutations when all Objects are Distinguishable

How many 3-letter "words" can you make with the letters MATH?

$\underline{4}\ \underline{3}\ \underline{2} = 24$

Investigate pg. 246 in groups

Communication | *Notation*

${}_nP_r$ is the notation commonly used to represent the number of permutations that can be made from a set of n different objects where only r of them are used in each arrangement, and $0 \le r \le n$.

When all available objects are used in each arrangement, n and r are equal, so the notation ${}_nP_n$ is used.

$${}_nP_r = \frac{n!}{(n-r)!}$$

9) $\underline{10} \cdot \underline{9}$

$= 90$

$= \frac{10!}{8!}$

$\frac{n!}{(n-r)!}$

EXAMPLE 1 Solving a permutation problem where only some of the objects are used in each arrangement

Matt has downloaded 10 new songs from an online music store. He wants to create a playlist using 6 of these songs arranged in any order. How many different 6-song playlists can be created from his new downloaded songs?

$\underline{10}\ \underline{9}\ \underline{8}\ \underline{7}\ \underline{6}\ \underline{5} = \frac{10!}{4!} = 151\ 200$

$${}_{10}P_6 = \frac{10!}{(10-6)!} = \frac{10!}{4!} = 151\ 200$$

Your Turn

Determine all the possible 7-song playlists, then 8-song playlists, and finally 9-song playlists that you can create from 10 songs. How does the value of ${}_nP_r$ change as r gets closer to n? Is this what you would have predicted? Explain.

${}_{10}P_7 = 604\ 800$

${}_{10}P_8 = 1\ 814\ 400$

${}_{10}P_9 = 3\ 628\ 000$

${}_{10}P_{10} = 10! = 3\ 628\ 000$

increases... since there are more options to choose from.

EXAMPLE 2 Defining 0!

Use the formula for ${}_nP_r$ to show that $0! = 1$.

if we choose 10 songs from 10 we know the answer is 10!

or ${}_{10}P_{10} = \frac{10!}{(10-10)!} = \frac{10!}{0!}$

since we can't ÷ by 0

$0! = 1$

Your Turn

State the values of n for which each expression is defined, where $n \in I$.

a) $(n + 3)!$

$n+3 \geq 0$

$\boxed{n \geq -3}$

b) $\frac{n!}{(n+2)!}$

$\boxed{n \geq 0}$

$n+2 \geq 0$

$\boxed{\cancel{n \geq -2}}$

only consider the restriction that includes both

EXAMPLE 3 Solving a permutation problem involving cases

Tania needs to create a password for a social networking website she registered with. The password can use any digits from 0 to 9 and/or any letters of the alphabet. The password is case sensitive, so she can use both lower- and upper-case letters. A password must be at least 5 characters to a maximum of 7 characters, and each character can be used only once in the password. How many different passwords are possible?

letters $= 26 \times 2$
$= 52$
$\# = \cancel{10}$ 62

case 1: 5 characters
${}_{62}P_5 = 776\,520\,240$

case 2: 6 chars.
${}_{62}P_6 = 4.426 \times 10^{10}$

case 3: 7 chars
${}_{62}P_7$
$= 2.479 \times 10^{12}$

$\text{Total} = {}_{62}P_5 + {}_{62}P_6 + {}_{62}P_7$

EXAMPLE 4 Solving a permutation problem with conditions

At a used car lot, seven different car models are to be parked close to the street for easy viewing.

a) The three red cars must be parked so that there is a red car at each end and the third red car is exactly in the middle. How many ways can the seven cars be parked?

b) The three red cars must be parked side by side. How many ways can the seven cars be parked?

a) 3 4 3 2 2 1 1
$= 3!\,4!$
$= 144$

b) let's create a "block" containing the 3 red cars

3! ways of arranging the red cars

$\boxed{3!}$ 4 3 2 1
5

and 5! ways of arranging the 5 spots

$\therefore 3!\,5! = 720$

Your Turn

How many ways can the seven cars be parked if the three red cars must be parked side by side and the other four cars must also be side by side?

$\boxed{\_\ 3!\ \_}$ $\boxed{\_\ \_\ 4!\ \_}$ $= 3! \cdot 4! \cdot 2!$

2! ways of arranging the blocks of cars $= 288$

EXAMPLE 5 Comparing arrangements created with and without repetition

A social insurance number (SIN) in Canada consists of a nine-digit number that uses the digits 0 to 9. If there are no restrictions on the digits selected for each position in the number, how many SINs can be created if each digit can be repeated? How does this compare with the number of SINs that can be created if no repetition is allowed?

With repeats

$10 \cdot 10 \cdot 10 \ldots = 10^9$

without repeats

${}_{10}P_9 = \frac{10!}{1!} = 3\,628\,800$

way more options

Your Turn

a) In reality, the Canadian government does not use 0, 8, or 9 as the first digit when assigning SINs to citizens and permanent residents, and repetition of digits is allowed. How many nine-digit SINs do not start with 0, 8, or 9?

b) SINs starting with the digit 9 are issued to temporary residents. How many SINs are there in total?

a) $\underline{7}\ \underline{9}\ \underline{9}\ \underline{9}\ \underline{9}\ \underline{9}\ \underline{9}\ \underline{9}\ \underline{9} = 7 \cdot 9^8$

not 0, 8, 9

$= 301\,327\,047$

b) $\underline{1} \cdot 9^8 = 43\,046\,721$

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