INVESTIGATE the Math

Tom is putting away the dishes after supper. In addition to putting away the bowls, cups, and cutlery, he has to stack seven dinner plates. Three of the plates are white and identical, while the remaining four plates are red, green, yellow, and blue.

How many different ways can he stack the seven plates to store them in the cupboard?

A. If all seven plates are different, in how many ways could he stack the plates? $_7P_7 = 7!$

B. Tom decided to think about the plates as if they are different. To do this, he represented the three identical white plates using three different letter codes: W_1, W_2, and W_3. He then used R for the red plate, G for the green plate, Y for the yellow plate, and B for the blue plate. List all the ways the plates could be stacked if the three white plates are stacked on top of the four coloured plates, and the four coloured plates are stacked in the order red, green, yellow, blue.

$3!$

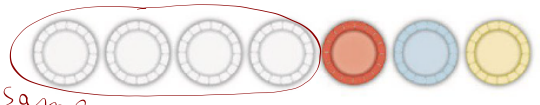
C. Examine your list in part B. Recognizing that the white plates are, in fact, identical, how many arrangements in your list are really the same arrangement? How does this number relate to the number of white plates?

1

D. Use your answers for parts A and C to write an expression that represents the number of different ways these seven plates can be stacked. Use your expression to calculate the number.

$\frac{7!}{3!}$

E. Suppose Tom had to stack these plates:

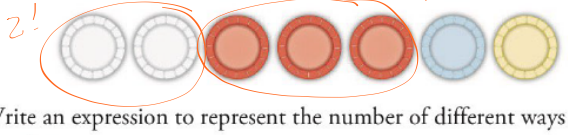


$4!$ are the same

Write an expression to represent the number of different ways to stack the plates.

$\frac{7!}{4!} = 7 \cdot 6 \cdot 5 = 210$

F. Suppose Tom had to stack these plates:



$3!$ $2!$

Write an expression to represent the number of different ways to stack the plates.

$\frac{7!}{3!2!} = 70$

G. The next time Tom puts away the dishes, the number of arrangements that are possible when he stacks the plates is represented by the expression

$$\frac{10!}{2! \cdot 3! \cdot 4!}$$

10 plates (2W, 3R, 4B, 1Y)

distinct colours

How many plates will he be stacking and what colours might they be?

H. Write an expression to represent the number of permutations of n objects, where a of the objects are identical, another b are identical, and another c are identical.

$\frac{n!}{a!b!c!}$

4.4 Permutations when all Objects are Identical

$4!$

How many 4-letter "words" can you make with the letters BIKE? And BOOK? $\frac{4!}{2!}$

EXAMPLE 1 Solving a permutation problem where objects are alike

In the mountainous regions of India, China, Nepal, and Bhutan, it is common to see prayer flags. Each flag has a prayer written on it, and colour is used to symbolize different elements: green (water), yellow (earth), white (air/wind), blue (sky/space), and red (fire).

How many different arrangements of the same prayer can Dorji make using these 9 flags: 1 green, 1 yellow, 2 white, 3 blue, and 2 red?

9 elements $\Rightarrow \frac{9!}{2!\,3!\,2!} = 15120$ different arrangements

repeat colours

Your Turn

Suppose there are 9 flags, but 3 are white, 3 are red, and 3 are green. Predict whether there would be more or fewer than 15 120 arrangements of these flags. State any assumptions you are making. Verify your prediction by determining the number of arrangements.

$\frac{9!}{3!3!3!} = 1680$

EXAMPLE 2 Solving a conditional permutation problem involving identical objects

How many ways can the letters of the word CANADA be arranged, if the first letter must be N and the last letter must be C?

how many other letters? $= 4$ $\therefore \frac{4!}{3!} = 4$ distinct arrangements

Your Turn

How many different arrangements are there for the six letters in the word CANADA for each situation?

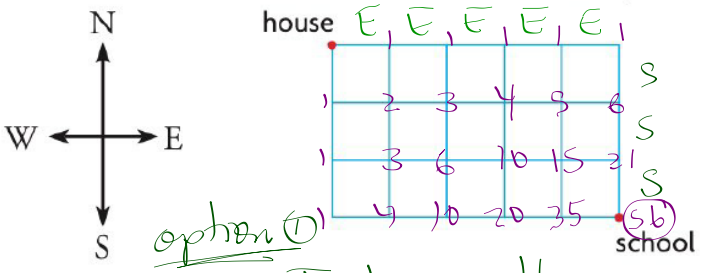
a) If there are no conditions for where letters must be placed $\frac{6!}{3!} = 120$

b) If the first letter has to be C $\frac{5!}{3!} = 20$

C · 5!

EXAMPLE 3 Solving a permutation problem involving routes

Julie's home is three blocks north and five blocks west of her school. How many routes can Julie take from home to school if she always travels either south or east?



option ①

As long as Julie walks East 5 blocks and South 3 blocks, she is guaranteed to get to school. Think about as a word with 8 letters

5 E's $\frac{8!}{5!3!} = 56$ pathways

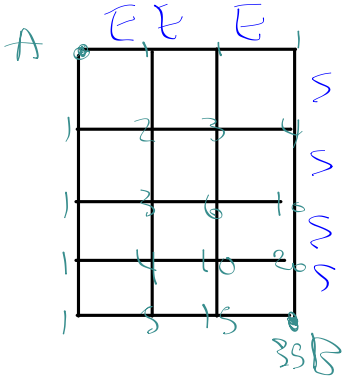
3 S's

option ②: Pascal's triangle

* Add the numbers of pathways coming from the North and West until you arrive at the destination

Your Turn

The school is three blocks east and four blocks south of Carrie's house. Predict whether Carrie will have more or fewer than 56 possible routes if she always travels south or east. Determine the number of routes to verify.



$\frac{7!}{3!4!} = 35$

Homework: pg. 266 #2, 5,-9, 11, 13, 15, 17, 18

Quiz tomorrow 4.1 - 4.3