4.6 Combinations

In how many ways can a team of three snow sculptors be chosen to represent Amir's school from the nine students who have volunteered?

EXAMPLE 1 Calculating combinations

Determine the number of three-person teams that can be formed from the nine volunteers.

$$\frac{9\cdot 8\cdot 7}{3!} = {}_9C_3 = \frac{9!}{(9-3)!\,3!} = 84$$

Communication Notation

${}_nC_r$ or $\binom{n}{r}$ are notations commonly used to represent the number of combinations that can be made from a set of n different objects where only r of them are used in each grouping, and $0 \leq r \leq n$.

${}_nC_r$ and $\binom{n}{r}$ are read as "n choose r."

EXAMPLE 2 Solving a simple combination problem

A restaurant serves 10 flavours of ice cream. Danielle has ordered a large sundae with three scoops of ice cream. How many different ice-cream combinations does Danielle have to choose from, if she wants each scoop to be a different flavour?

$$_{10}C_3 = \frac{10!}{7!\,3!} = \frac{10\cdot \overset{3}{\cancel{9}}\cdot \overset{4}{\cancel{8}}}{\cancel{3}\cdot\cancel{2}} = 120$$

Your Turn

Danielle's favourite flavour is chocolate. If one scoop in her large sundae must be chocolate and the other two must be different flavours, how many combinations of ice cream are possible?

$$_9C_2 = \frac{9\cdot 8}{2} = 36$$

EXAMPLE 3 Solving a combination problem using the Fundamental Counting Principle

Tanya is the coach of a Pole Push team that consists of nine players: five male and four female. In each competition, teams of four compete against each other to push their competitors out of a circle. The team that is successful wins.

a) How many different four-person teams does Tanya have to choose from for an all-male competition?

b) How many different four-person teams does Tanya have to choose from, with two males and two females, for a mixed competition?

a) $_5C_4 = \frac{5!}{4!1!} = 5$

b) boys × girls

$_5C_2 \times {}_4C_2$

$= 10 \times 6$

$= 60$ different mixed teams

Your Turn

How many different four-person teams does Tanya have to choose from for an all-women competition?

$_4C_4 = 1$

EXAMPLE 4 Solving a combination problem by considering cases

A planning committee is to be formed for a school-wide Earth Day program. There are 13 volunteers: 8 teachers and 5 students. How many ways can the principal choose a 4-person committee that has at least 1 teacher?

We could consider the cases with 1, 2, 3 or 4 teachers and add them up:

$_8C_1 \cdot {}_5C_3 + {}_8C_2 \cdot {}_5C_2 + {}_8C_3 \cdot {}_5C_1 + {}_8C_4 \cdot {}_5C_0$

one teacher, 3 students; 2 teachers, 2 students; 3 teachers, 1 student; only teachers

$(8)(10) + (28)(10) + (56)(5) + (70)(1)$

$80 + 280 + 280 + 70 = 710$ different committees with at least one teacher

Homework: pg. 280 #1, 2, 4, 5, 8, 9, 11, 13, 16, 17, 19

OR use the complement:

take away the # of ways of having NO teachers from all possible combinations.

All − No teachers

$= {}_{13}C_4 - {}_8C_0 \cdot {}_5C_4 = 715 - (1)(5) = 710$

Same!!