# 4.7 Solving Counting Problems

Explore pg. 283 together & Investigate pg. 283 in partners

A) order matters

B) H 6 songs H 6 songs P

C) 1 D) 2 E) 1

F) $_{12}P_{12} = 12!$

G) $_2P_2 \times {}_{12}P_{12} = 2!\,12!$

$= 958\,003\,200$

H) permutations

I) could have

J) $2!\,13!$ — 13 x more options

## EXAMPLE 1 Solving a permutation problem with conditions

A piano teacher and her students are having a group photograph taken. There are three boys and five girls. The photographer wants the boys to sit together and the girls to sit together for one of the poses. How many ways can the students and teacher sit in a row of nine chairs for this pose?

Boys | Girls | teacher

$3!$ | $5!$ | $1!$

$= (3!\,5!\,1!)\,3!$

3! ways of ordering these elements

$= 4320$

## Your Turn

For another pose, the photographer wants the two tallest students, Jill and Sam, to sit at either end, Jill on the left and Sam on the right, and the teacher to sit in the middle. How many different seating arrangements are there for this pose?

J 6·5·4·I·3·2·1 S $= 6!$

$= 720$

**EXAMPLE 2** Solving a combination problem involving multiple choices

Combination problems are common in computer science. Suppose there is a set of 10 different data items represented by {a, b, c, d, e, f, g, h, i, j} to be placed into four different memory cells in a computer. Only 3 data items are to be placed in the first cell, 4 data items in the second cell, 2 data items in the third cell, and 1 data item in the last cell. How many ways can the 10 data items be placed in the four memory cells?

AND

$${}_{10}C_3 \cdot {}_{7}C_4 \cdot {}_{3}C_2 \cdot {}_{1}C_1 = 12\,600$$

7 data left

now only 3 data left

**EXAMPLE 3** Solving a combination problem involving cases

How many different five-card hands that contain at most one black card can be dealt to one person from a standard deck of playing cards?

addition

At most means: NO black OR 1 black

$$= {}_{26}C_5 + {}_{26}C_1 \cdot {}_{26}C_4$$

# of reds

# I'm choosing

one black AND 4 reds

$$= 65\,780 + 388\,700 = 454\,480$$

***Your Turn***

Solve the problem above using indirect reasoning. To do this, you will need to consider the total number of five-card hands as well as the number of five-card hands that do not meet the condition "at most one black card."

All combos − (2 black + 3 black + 4 black + 5 blacks)

$${}_{52}C_5 - \left[{}_{26}C_2 \cdot {}_{26}C_3 + {}_{26}C_3 \cdot {}_{26}C_2 + {}_{26}C_4 \cdot {}_{26}C_1 + {}_{26}C_5\right]$$

$$2\,598\,960 - \left[845\,000 + 845\,000 + 388\,700 + 65\,780\right]$$

Homework: pg. 288 #1, 2, 5, 6, 7, 9, 10, 11, 13, 14, 16, 17

$$2\,598\,960 - 2\,144\,480 = \boxed{454\,480}$$ 5