$\frac{1}{x}$

What value of x do you need to make its reciprocal...

Positive? $\longrightarrow$ $x > 0$ $\quad \frac{1}{+} = +$

Negative? $\longrightarrow$ $x < 0$ $\quad \frac{1}{-} = -$

Two? $\longrightarrow$ $\frac{1}{?} = 2$ $\quad ? = 0.5$

One? $\longrightarrow$ $\frac{1}{1} = 1$

Negative one? $\longrightarrow$ $\frac{1}{-1} = -1$

$\therefore$ the reciprocal of 1 and −1 are unchanged ... invariant!

Negative ten? $\longrightarrow$ $\frac{1}{?} = -10 \Rightarrow ? = -0.1$ or $-\frac{1}{10}$

One half? $\longrightarrow$ $\frac{1}{?} = \frac{1}{2} \Rightarrow ? = 2$

Zero? $\longrightarrow$ $\frac{1}{?} = 0 \Rightarrow$ not possible

Undefined? $\longrightarrow$ $\frac{1}{?} =$ undefined $\Rightarrow$ zero

Chapter 5: Rational Functions

5.1 Reciprocals of Polynomial Functions

Examples of Reciprocal Polynomial Functions:

$f(x) = \frac{1}{x}$ $g(x) = \frac{1}{x^2+7}$ $h(x) = \frac{1}{x^3+7x^2-0.7}$

We can create a graph of a reciprocal polynomial function by examining the graph of the polynomial.

	Original Polynomial	*Reciprocal Function*
y - intercepts	$(0, q)$	$(0, 1/q)$
zeroes	$x = s, x = t \ldots$	V.A. @ $x = s, x = t, \ldots$
domain	$x \in \mathbb{R}$ $\to$	$x \neq s, t, \ldots$
Positive Values		stay positive
Negative Values		stay negative
Interval of Decrease	$\to$	interval of increase
Interval of Increase	$\to$	interval of decrease
The "Ones"	$f(a) = \pm 1$	$\frac{1}{f(a)} = \pm 1$

invariant points

End behaviour:

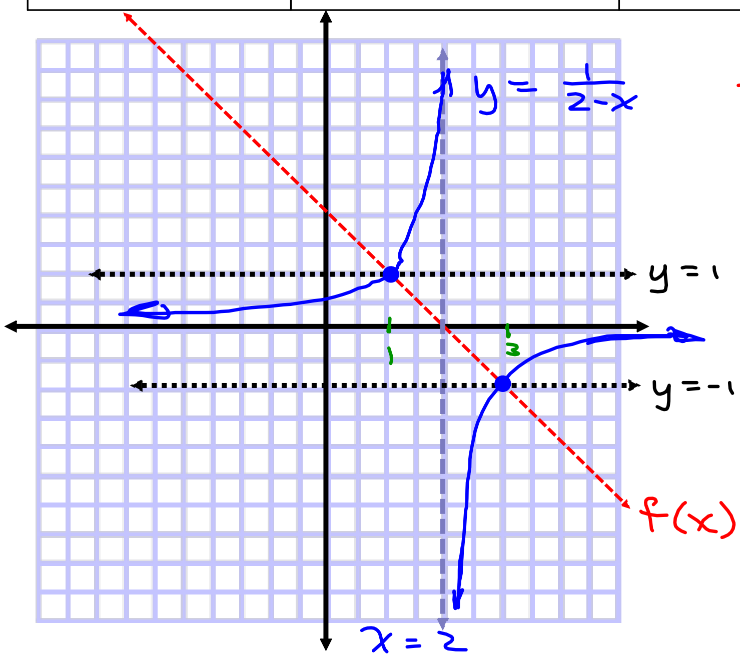
as $y \to \infty$ $y \to 0^+$ from above (+ y-value)

$y \to -\infty$ $y \to 0^-$ from below

these statements are reversible

Example 1: For the function f(x) = 2 - x, describe the function's graph and then sketch the reciprocal function.

	Original Polynomial	*Reciprocal Function*
y - intercepts	$(0,2)$	$(0, \frac{1}{2})$
zeroes	$x = 2$	V.A. @ $x = 2$
domain	$x \in \mathbb{R}$	$x \neq 2$
Positive Values	$(-\infty, 2)$ →	same
Negative Values	$(2, \infty)$ →	same
Interval of Decrease	$x \in \mathbb{R}$	none
Interval of Increase	none	all except $x = 2$ $(-\infty, 2) \cup (2, \infty)$
The "Ones"	$(1,1)$ & $(3,-1)$ →	stay same



$f(x) = 2 - x$

$y = \frac{1}{2-x}$

for invariants:

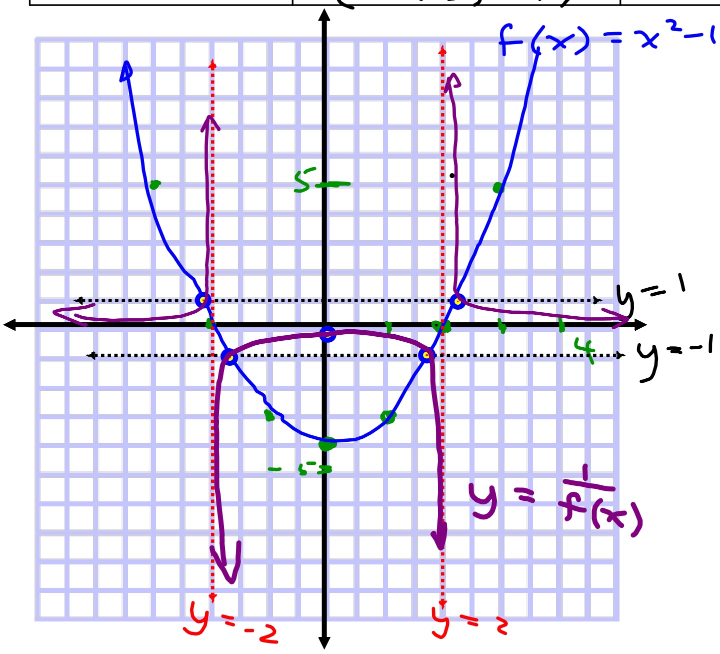
set $f(x) = \pm 1$

$1 = 2 - x$ $\quad$ $-1 = 2 - x$

$\boxed{x = 1}$ $\quad$ $\boxed{x = 3}$

Example 2: Describe the graph of $f(x) = x^2 - 4$ and then sketch its reciprocal graph

	Original Polynomial	*Reciprocal Function*
y - intercepts	$(0,-4)$	$(0,-\frac{1}{4})$
zeroes	$x = \pm 2$	V.a @ $x = \pm 2$
domain	$x \in \mathbb{R}$	$\Rightarrow x \neq \pm 2$
Positive Values	$(-\infty,-2) \cup (2,\infty)$	$\Rightarrow$ same
Negative Values	$(-2,2)$	$\Rightarrow$ same
Interval of Decrease	$(-\infty, 0)$	$(0,\infty)$ $x \neq 2$
Interval of Increase	$(0, \infty)$	$(-\infty,0)$ $x \neq -2$
The "Ones"	$(\pm\sqrt{5}, 1)$ $(\pm\sqrt{3}, -1)$	$\Rightarrow$



the invariant pts:

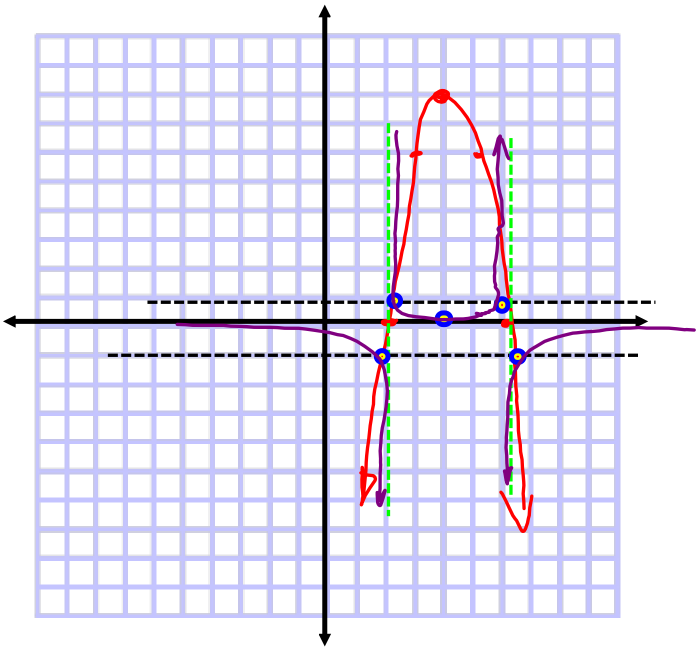
$f(x) = \pm 1$

$1 = x^2 - 4$	$-1 = x^2 - 4$
$5 = x^2$	$3 = x^2$
$x = \pm\sqrt{5}$	$x = \pm\sqrt{3}$

Example 3: For the function $g(x) = -2(x - 4)^2 + 8$, sketch its reciprocal function.

vertex @ (4, 8)

	Original Polynomial	*Reciprocal Function*
y - intercepts		
zeroes		
domain		
Positive Values		
Negative Values		
Interval of Decrease		
Interval of Increase		
The "Ones"		



$1 \cdot 2 = 2$
$3 \cdot 2 = 6$
$5 \cdot 2 = 10$
$7 \cdot 2 = 14$

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