5.2 Quotients of Polynomial Functions

Rational function: if it can be expressed as

$$f(x) = \frac{p(x)}{q(x)}, \textit{if } p(x) \,\&\, q(x)$$ are both polynomial functions

and $q(x) \neq 0$

Example:

$$y = \frac{2x+1}{x^2-5} \qquad y = \frac{3}{x}$$

Counterexample:

$$y = \frac{3x^2+1}{\sqrt{x-1}}$$

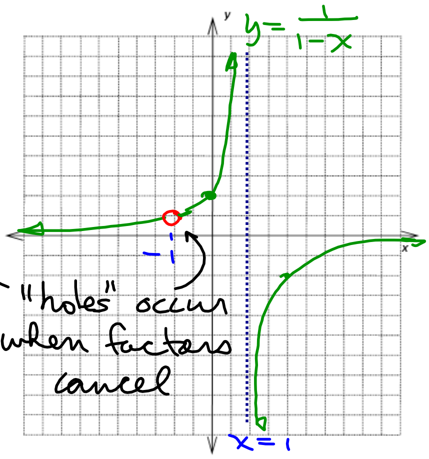
not a polynomial

$$\sqrt{x-1} = (x-1)^{1/2}$$

not a natural number

Pg. 258 EXPLORE THE MATH

A. *If* $f(x) = x+1$ *and* $g(x) = 1 - x^2$ *graph* $y = \frac{f(x)}{g(x)}$



$y = \frac{\cancel{x+1}}{(1-x)\cancel{(1+x)}} = \frac{1}{1-x}$

$x \neq \pm 1$

or $= \frac{1}{-(x-1)}$

V.A @ $x = 1$

"holes" occur when factors cancel

@ $x = -1$ we have a "hole discontinuity"

i) zeroes come from the numerator

so... $0^{(1-x^2)} = \frac{x+1}{\cancel{1-x^2}}\cancel{(1-x^2)}$

$0 = x+1$

$\boxed{-1 = x}$

should yield a zero but here it's a hole...

↳ so no zero!

ii) vertical asymptotes when denominator = 0

so... $1 - x^2 = 0$

$1 = x^2$

$x = \pm 1$

but $(1+x)$ cancels so...

V.A @ $1 - x = 0$

x

iii) D: $\{x \in \mathbb{R} \mid x \neq \pm 1\}$

R: has a discontinuity for $x = -1$ so use the simplified equ^n to get the discontinued y-value

$y = \frac{1}{1-x}$

$y = \frac{1}{1-(-1)} = \frac{1}{2}$

R: $\{y \in \mathbb{R} \mid y \neq \frac{1}{2}, 0\}$

iv) end behaviour... consider limits

as $x \to \infty$

$y = \frac{\cancel{\infty}+1}{1-(\infty)^{\cancel{2}}}$

$= \frac{\cancel{\infty}}{-(\infty)^{\cancel{2}}}$

$= \frac{1}{-\infty}$

$= 0^-$

as $x \to -\infty$

$y = \frac{-\cancel{\infty}+1}{1-(-\infty)^{\cancel{2}}}$

$= \frac{1}{\infty}$

$= 0^+$

since both limits approach the same value... $y = 0$, we say there is a horizontal asymptote with equ^n $\boxed{y=0}$

v) is $\frac{f(x)}{g(x)}$ a function?

$\Rightarrow$ yes it passes the VLT.

C. graph $y = \frac{g(x)}{f(x)} = \frac{1-x^2}{x+1}$

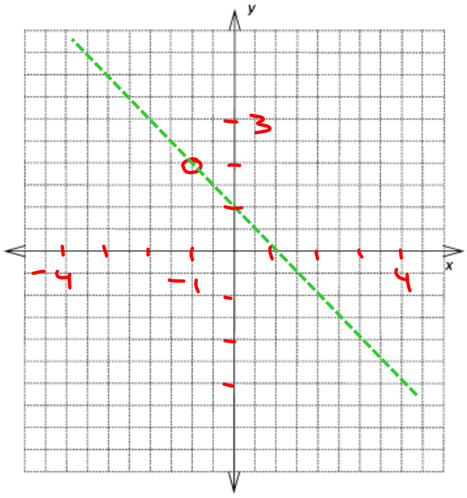
$= \frac{(\cancel{1+x})(1-x)}{\cancel{x+1}}$

$= 1 - x$ looks like

with a hole for

$1 + x = 0$

$\boxed{x = -1}$



what are the (x, y) of the hole?

$\Rightarrow$ use simplified eqn $\Rightarrow$ $y = 1 - x$
with $x = -1$

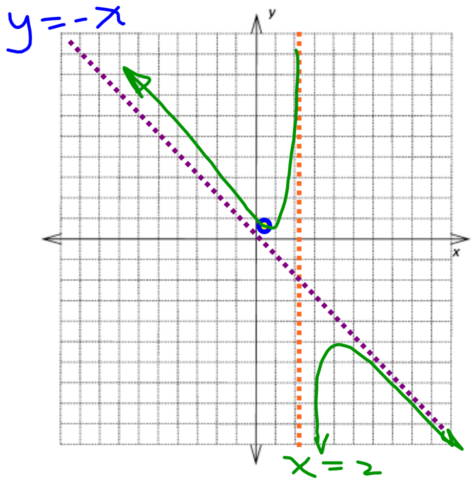
$= 1 - (-1)$

$= 2$

$\therefore$ hole @ $(-1, 2)$

D: $\{x \in \mathbb{R} \mid x \neq -1\}$

R: $\{y \in \mathbb{R} \mid y \neq 2\}$



E. *graph* $y = \frac{1+x^2}{2-x}$

Zeroes: $0 = 1 + x^2$

$-1 = x^2$

$\therefore$ no real zeroes

discontinuity when $2 - x = 0$

V.A. $\rightarrow$ $\boxed{x = 2}$

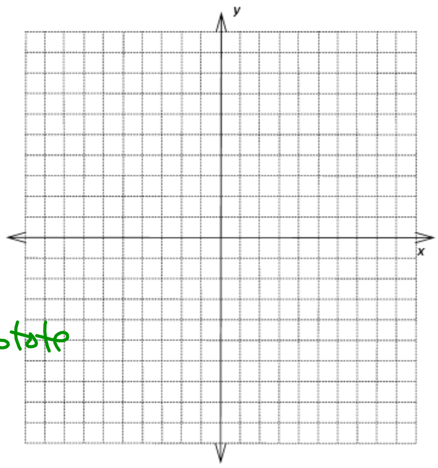
* E.B. as $x \to \infty$ as $x \to -\infty$

$y = \frac{1+\infty^2}{2-\infty}$ $y = \frac{1+(-\infty)^2}{2-(-\infty)}$

$= -\infty$ $= \infty$

high to low

~~graph~~ $\cancel{y = \frac{2-x}{1+x}}$



but consider the limits

$y = \lim_{x \to \infty} \frac{1+x^2}{2-x} = \frac{x^{\cancel{2}}}{-\cancel{x}} = -x$

$\therefore$ $\boxed{y = -x}$ is the equ^n of the oblique asymptote

* y-int: $x = 0$

$y = \frac{1+(0)^2}{2-(0)} = \frac{1}{2} \rightarrow (0, \frac{1}{2})$

* check behaviour close to V.A.

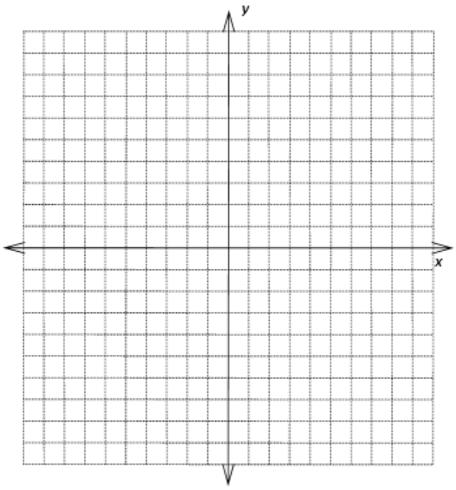
test $x = 2.1$ and

$x = 1.9$

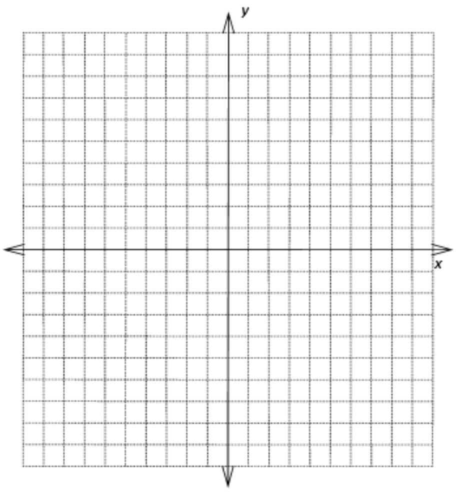
* oblique asymptotes occur when the degree of the numerator is exactly one more than that of the denominator.

* a horizontal asymptote occurs when the degree of the denominator $\geq$ that of the numerator

E. $graph\ y = \frac{x-1}{x}$

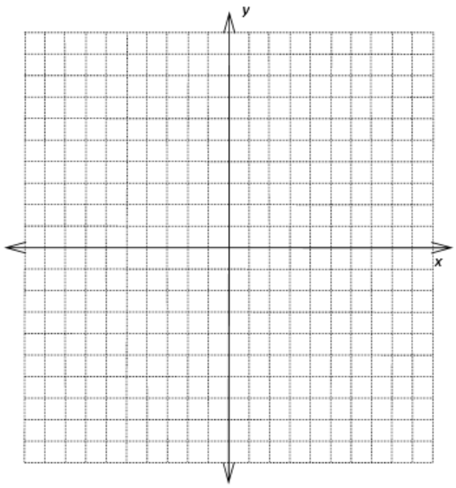


$graph\ y = \frac{x-1}{x}$



J. Horizontal asymptotes

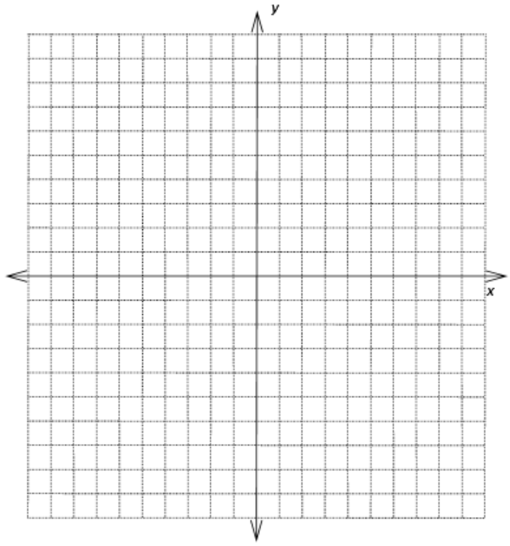
F. $graph\ y = \dfrac{x^2 - 1}{x - 1}$



G. “holes” vs. vertical asymptotes

H. Oblique asymptotes

$$graph\ y = \frac{x^2 + 2x}{x+1}$$



See summary on Pg. 261

Homefun: Pg. 262 #1-3