

5.3 Equations avec radicaux

* Des restrictions sur une variable existe quand:

- > Le dénominateur est zéro
- > Le radicande est négatif pour des indices paires

ex. Indique les restrictions pour:

$$a) \frac{3}{2x-8} = \sqrt{2x-5}$$

$$\begin{matrix} 2x \\ 2 \\ \cancel{x} \end{matrix} \neq 8$$

$$x \neq 4$$

$$b) \frac{1}{\sqrt{5-4x}} = 3x-5$$

Puisque $5-4x$ est
un radicande ET un
dénominateur ...

$$\begin{matrix} 5-4x \\ 5 \\ -4x \end{matrix} \geq 0$$

et

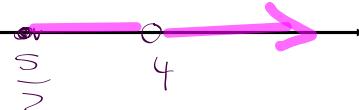
$$5-4x \neq 0$$

$$2x-5 \geq 0$$

$$\begin{matrix} 2x \\ 2 \\ x \end{matrix} \geq \frac{5}{2}$$

$$x \geq \frac{5}{2}$$

$$x \in \left[\frac{5}{2}, 4 \right] \cup (4, \infty)$$



* When solving equations containing radicals, the key is to isoler the radical on one side of the equation, then prend le carré to eliminate the radical.

$$\begin{matrix} 5-4x & \geq 0 \\ \frac{5}{4} & \geq \frac{4x}{4} \\ \frac{5}{4} & \geq x \end{matrix} \Rightarrow x \leq \frac{5}{4}$$

NOTE: When squaring both sides, we lose accuracy. Therefore, we must always verify our answers in the original equation.

$$ex. 5 + \sqrt{2x+1} = 12$$

solution:

$$(\sqrt{2x+1})^2 = 7^2$$

$$2x+1 = 49$$

$$\begin{matrix} 2x \\ 2 \\ x \end{matrix} = \frac{48}{2}$$

$$x = 24$$

restrictions: $2x+1 \geq 0$

$$x \geq -\frac{1}{2}$$

vérification:

$$C.Q. = 5 + \sqrt{2(24)+1}$$

$$= 5 + \sqrt{48+1}$$

$$= 5 + 7$$

$$= 12 = C.D. \text{ yay!}$$

ex. Solve $n - \sqrt{5-n} = -7$

solution:

$$(n+7)^2 = (\sqrt{5-n})^2$$

$$(n+7)(n+7) = 5-n$$

$$n^2 + 14n + 49 = 5-n$$

$$n^2 + 15n + 44 = 0$$

$$(n+11)(n+4) = 0$$

$$\begin{array}{l} n+11=0 \\ n=-11 \end{array}$$

$$\begin{array}{l} n+4=0 \\ n=-4 \end{array}$$

ex. Solve $7 - \sqrt{3x} = \sqrt{5x+4} + 5$

solution:

$$(2)^2 = (\sqrt{5x+4} + \sqrt{3x})^2$$

$$4 = (\sqrt{5x+4} + \sqrt{3x})(\sqrt{5x+4} + \sqrt{3x})$$

$$4 = 5x+4 + 3x + 2\sqrt{3x(5x+4)}$$

$$4 = 8x+4 + 2\sqrt{15x^2+12x}$$

$$\frac{-8x}{2} = \frac{2\sqrt{15x^2+12x}}{2}$$

$$(-4x)^2 = (\sqrt{15x^2+12x})^2$$

$$16x^2 = 15x^2 + 12x$$

$$x^2 - 12x = 0$$

$$x(x-12) = 0$$

Recall: $x=0$ $x=12$

$$\text{si } x^2 = 3 \quad x^2 \leq 5$$

$$x = \pm\sqrt{3} \quad \text{aucune restriction}$$

$$\sqrt{x^2+1} \Rightarrow x^2+1 \geq 0 \quad \text{toujours vrai}$$

∴ aucune restriction

restrictions:

$$\begin{array}{l} 5-n \geq 0 \\ 5 \geq n \end{array}$$

vérification:

$$\boxed{n=-11}$$

$$C.G. = (-11) - \sqrt{5-(-11)}$$

$$= -11 - 4$$

$$= -15 \neq C.D.$$

$$\boxed{n=-4}$$

$$C.G. = (-4) - \sqrt{5-(-4)}$$

$$= -4 - \sqrt{9}$$

$$= -7 = C.D.$$

restrictions:

$$\begin{array}{l} 3x \geq 0 \quad 5x+4 \geq 0 \\ \boxed{x \geq 0} \quad \frac{5x}{5} \geq \frac{4}{5} \\ \boxed{x \geq \frac{4}{5}} \end{array}$$

vérification:

$$\begin{array}{l} C.G. = 7 - \sqrt{3(0)} \quad C.D. = \sqrt{5(0)+4} + 5 \\ = 7 \quad = \sqrt{4} + 5 \\ = 7 \end{array}$$

$$\boxed{x=12}$$

$$\begin{array}{l} C.G. = 7 - \sqrt{3(12)} \quad C.D. = \sqrt{5(12)+4} + 5 \\ = 7 - \sqrt{36} \quad = \sqrt{64} + 5 \\ = 1 \quad = 13 \end{array}$$

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(qd), 10c)