5.3 Equations containing radicals

* Restrictions on a variable exist when:

> The denominator is = zero

> The radicand is negative for even indices

ex. State the restrictions for:

a) $\frac{3}{2x-8} = \sqrt{2x-5}$

$2x-8 \neq 0$

$\frac{2x}{2} \neq \frac{8}{2}$

$\boxed{x \neq 4}$

$2x-5 \geq 0$

$\frac{2x}{2} \geq \frac{5}{2}$

$\boxed{x \geq \frac{5}{2}}$

b) $\frac{1}{\sqrt{5-4x}} = 3x-5$

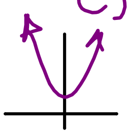
since $5-4x$ is both a radicand AND a denominator

$5-4x \geq 0$ $\quad$ $5-4x \neq 0$

combine

$\boxed{5-4x > 0}$

c) $5 = \sqrt{x^2+1}$



$x^2+1 \geq 0$

$x^2 \geq -1$

since x^2 is always $\oplus$, there are restrictions

* When solving equations containing radicals, the key is to isolate the radical on one side of the equation, then square both sides to eliminate the radical.

NOTE: When squaring both sides, we lose accuracy. Therefore, we must always verify our answers in the original equation.

ex. $5 + \sqrt{2x + 1} = 12$

solution:

$(\sqrt{2x+1})^2 = (7)^2$

$2x+1 = 49$

$\frac{2x}{2} = \frac{48}{2}$

$\boxed{x = 24}$ ✓

satisfies my restriction

restrictions: $2x+1 \geq 0$

$\frac{2x}{2} \geq \frac{-1}{2}$

$\boxed{x \geq -\frac{1}{2}}$

verification:

$LS = 5 + \sqrt{2(24)+1}$

$= 5 + \sqrt{49}$

$= 5 + 7$

$= 12$

$= RS$

$\therefore$ since $LS = RS$, $x = 24$

ex. Solve $n - \sqrt{5 - n} = -7$

solution:

$(n+7)^2 = (\sqrt{5-n})^2$

$(n+7)(n+7) = 5-n$

$n^2+14n+49 = 5-n$

$n^2+15n+44=0$

$4 \cdot 11 = 44$

$4 + 11 = 15$

$(n+4)(n+11)=0$

$\boxed{n=-4}$ $\boxed{\cancel{n=-11}}$

restrictions: $5-n \geq 0$

$\boxed{5 \geq n}$

verification:

$\boxed{n=-4}$	$\boxed{\cancel{n=-11}}$
$LS=(-4)-\sqrt{5-(-4)}$	$LS=(-11)-\sqrt{5-(-11)}$
$=-4-\sqrt{9}$	$=-11-\sqrt{16}$
$=-4-3$	$=-11-4$
$=-7$	$=-15$
$=RS$	$\neq RS$

ex. Solve $7 - \sqrt{3x} = \sqrt{5x + 4} + 5$

solution:

$(2)^2 = (\sqrt{5x+4}+\sqrt{3x})^2$

$4=(\sqrt{5x+4}+\sqrt{3x})(\sqrt{5x+4}+\sqrt{3x})$

$4 = 5x+4+3x+2\sqrt{3x(5x+4)}$

$\frac{-8x}{2} = \frac{2\sqrt{15x^2+12x}}{2}$

$(-4x)^2 = (\sqrt{15x^2+12x})^2$

$16x^2 = 15x^2+12x$

$x^2-12x=0$

$x(x-12)=0$

$\boxed{x=0}$ $\boxed{\cancel{x=12}}$

dominant restriction

restrictions:

$3x \geq 0$

$\boxed{x \geq 0}$

$5x+4 \geq 0$

$\frac{5x}{5} \geq \frac{-4}{5}$

$\boxed{\cancel{x \geq -4/5}}$

$-4/5$ 0

verification:

$\boxed{x=0}$	$\boxed{\cancel{x=12}}$
$LS=7-\sqrt{3(0)}$	$LS=7-\sqrt{3(12)}$
$=7$	$=7-\sqrt{36}$
$RS=\sqrt{5(0)+4}+5$	$=1$
$=\sqrt{4}+5$	$RS=\sqrt{5(12)+4}+5$
$=7$	$=\sqrt{60+4}+5$
$LS=RS$	$=8+5$
	$=13$
	$LS \neq RS$

Recall:

$x^2=3$
$\boxed{x=\pm\sqrt{3}}$

$x^2=0$
$\boxed{x=0}$

$x^2=-5$
$\boxed{\text{no sol'n}}$

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Test in 2 days