5.3 Graphing Rational Functions

Steps

Factor top and bottom... from this determine zeroes (top) and V.A.'s (bottom) or holes
Find y-intercept (plug x = 0)
Determine end behaviour (H.A. or O.A.)
Use test points on either side of V.A.'s and zeroes to get shape of curve

Example: Graph $f(x)=\dfrac{x^2-5x-6}{x^2-4}$

$f(x)=\dfrac{(x-6)(x+1)}{(x+2)(x-2)}$

V.A @ $x=\pm 2$

H.A @ $y=\dfrac{x^2}{x^2} \Rightarrow \boxed{y=1}$

y-int : $x=0$

$y=\dfrac{-6}{-4}=\dfrac{3}{2}$

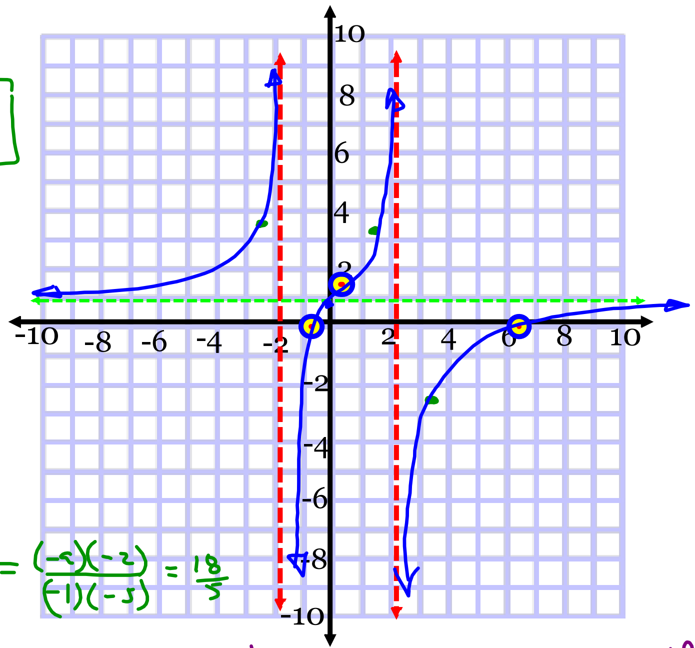
zeroes: $0=\dfrac{(x-6)(x+1)}{(x+2)(x-2)}$

$\boxed{x=6}$ $\boxed{x=-1}$

$f(3)=\dfrac{(-3)(4)}{(5)(1)}=-\dfrac{12}{5}$

$f(1)=\dfrac{(-5)(2)}{(3)(-1)}=\dfrac{10}{3}$

$f(-3)=\dfrac{(-9)(-2)}{(-1)(-5)}=\dfrac{18}{5}$



where f(x) crosses the H.A.

$1=\dfrac{(x-6)(x+1)}{(x+2)(x-2)}$

$\cancel{x^2}-4=\cancel{x^2}-5x-6$

$5x=-2 \Rightarrow x=-2/5$

Example: Graph $f(x)=\dfrac{x^2-x-12}{x+2}$

$f(x)=\dfrac{(x-4)(x+3)}{x+2}$

V.A @ $\boxed{x=-2}$

H.A. @ $y=\dfrac{x^{\cancel{2}}}{\cancel{x}} \Rightarrow \boxed{y=x}$

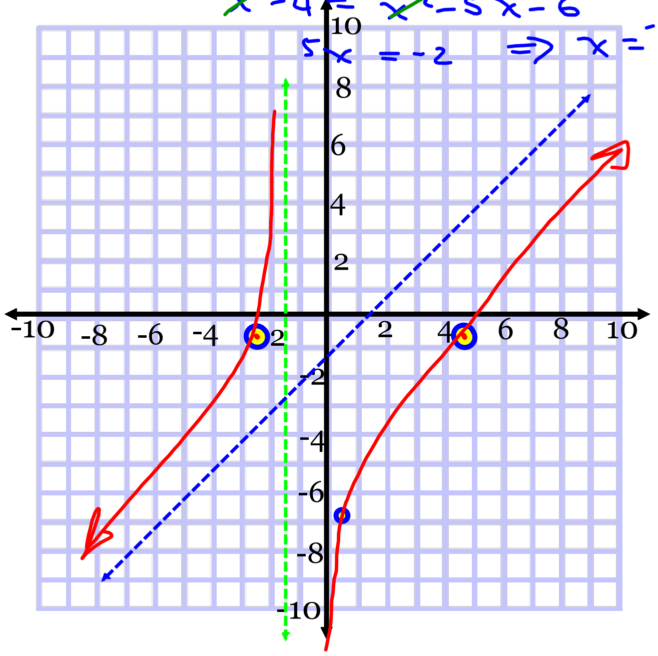
y-int: $x=0$

$y=\dfrac{-12}{2}=-6$

x-int: $y=0$

$0=(x-4)(x+3)$

$\boxed{x=4,-3}$



Example: Graph $f(x) = \frac{x+5}{x^3+7x^2+10x} = \frac{\cancel{x+5}}{x(x+2)\cancel{(x+5)}}$

$g(x) = \frac{1}{x(x+2)}$

hole @ $x=-5$

hole @ $x=-5$

$g(-5) = \frac{1}{-5(-5+2)} = \frac{1}{15}$

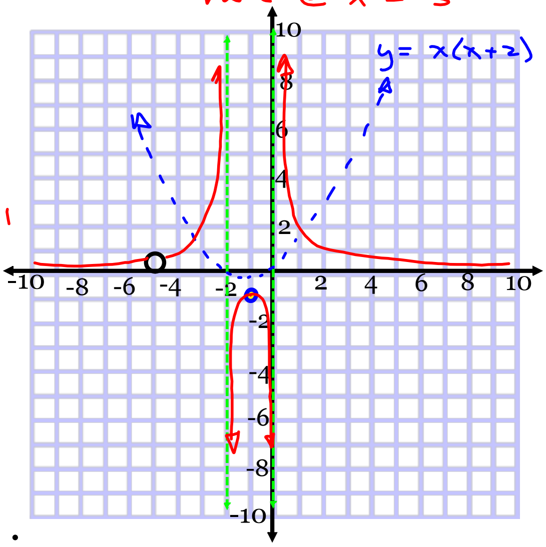
V.A @ $x=0,-2$

$f(-1) = \frac{1}{(-1)(-1+2)} = \frac{1}{-1} = -1$

H.A. $y = \frac{x}{x^3} = \frac{1}{x^2}$

but as $x \to \infty$

$y \to 0$



Example: Graph f(x) = ~~(2x² - 8)/(x² - 9)~~ $\frac{4x+3}{x-1}$ ⇒ looks like $\frac{1}{x}$?

H.A. @ $y = \frac{4\cancel{x}}{\cancel{x}}$ $\boxed{y=4}$

V.A @ $\boxed{x=1}$

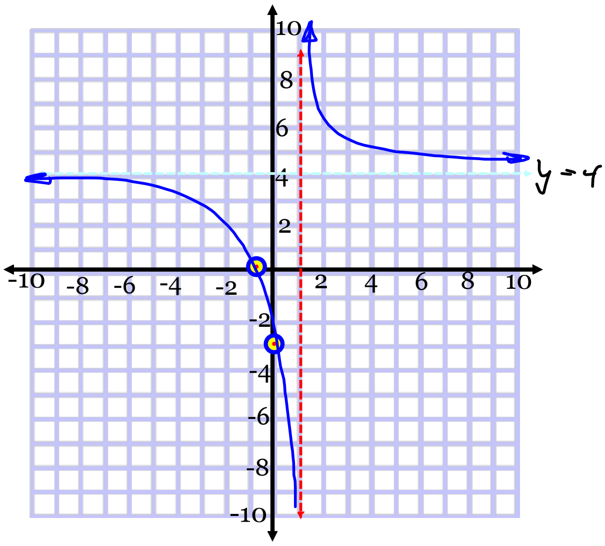
y-int: $y = \frac{4(0)+3}{0-1}$

$\boxed{y=-3}$

x-int: $y=0$

$0 = 4x+3$

$\boxed{x = -3/4}$



$y = \frac{4x+3}{x-1}$

$= \frac{[4x+3-3x-4]+3x+4}{x-1}$

$= \frac{(x-1)}{x-1} + \frac{(3x+4)}{x-1}$

$= 1 + \frac{3x+4}{x-1}$

continue on this way and get

$= 4 + \frac{1}{x-1}$

looks like $y=\frac{1}{x}$ moved up 4 units and right 1 unit

Homefun

Pg. 272 #1-3, 8, 10, 12, 13