5.3 Probability Using Counting Methods

Explore pg. 313 together

Investigate pg. 313 together

A. yes

B. no, any order is acceptable

C. $_{10}C_3 = 120$

D. $_9C_2 = 36$

9 left, we need 2 more

E. $P(V) = \frac{36}{120} = 30\%$

F. replacement names go back in the hat each draw → no replacement

G. more likely

H. less likely

EXAMPLE 1 Solving a probability problem using counting techniques

Jamaal, Ethan, and Alberto are competing with seven other boys to be on their school's cross-country team. All the boys have an equal chance of winning the trial race. Determine the probability that Jamaal, Ethan, and Alberto will place first, second, and third, in any order.

10 - boys

→ C not P

$n(J, E, A) = {}_3C_3 = 1$

$n(\text{all outcomes}) = {}_{10}C_3 = 120$

$P(J, E, A) = \frac{1}{120}$

Your Turn

Suppose that Zachary is also trying out for the team, so now there will be 11 runners in the trial race. What is the probability that three of Jamal, Ethan, Alberto, and Zachary will place in the top three positions?

$n(\text{all}) = {}_{11}C_3$

$n(J, E, A \text{ or } Z) = {}_4C_3$

$P(3 \text{ of } 4) = \frac{{}_4C_3}{{}_{11}C_3} = \frac{4}{165}$

EXAMPLE 2 Solving a probability problem with the Fundamental Counting Principle

About 20 years after they graduated from high school, Blake, Mario, and Simon met in a mall. Blake had two daughters with him, and he said he had three other children at home. Determine the probability that at least one of Blake's children is a boy.

Consider P(no boys)

n(no boys) = 1 → Girl, Girl, Girl

n(3 kids) = 2 · 2 · 2 = 8

P(no boys) = 1/8

P(no boys') = 7/8

Your Turn

Suppose that Blake had had one daughter with him at the mall and four children at home. Determine the probability of each event.

a) All five of Blake's children are girls.

b) At least one of Blake's children is a boy.

a) $P(4G) = \frac{1}{16}$

b) $P(4G') = P(\text{at least one B}) = \frac{15}{16}$

EXAMPLE 3 Solving a probability problem using reasoning

Beau hosts a morning radio show in Saskatoon. To advertise his show, he is holding a contest at a local mall. He spells out SASKATCHEWAN with letter tiles. Then he turns the tiles face down and mixes them up. He asks Sally to arrange the tiles in a row and turn them face up. If the row of tiles spells SASKATCHEWAN, Sally will win a new car. Determine the probability that Sally will win the car.

$n(\text{spell}) = \frac{12!}{2!3!} = 19\,958\,400$

$P(\text{correct}) = \frac{1}{19\,958\,400}$

EXAMPLE 4 Solving a probability problem with conditions

There are 18 bikes in Marnie's spinning class. The bikes are arranged in 3 rows, with 6 bikes in each row. Allison, Brett, Carol, Doug, Erica, and Franco each call the gym to reserve a bike. They hope to be in the same row, but they cannot request a specific bike. Determine the probability that all 6 friends will be in the same row, with Allison and Franco at either end.

$n(\text{satisfy}) = 3 \cdot {}_2P_2 \cdot {}_4P_4 \cdot {}_{12}P_{12}$

(3 = # rows; ${}_2P_2$ = A and F; ${}_4P_4$ = 4 others; ${}_{12}P_{12}$ = strangers)

$n(\text{all}) = {}_{18}P_{18}$

$$P(\text{satisfy}) = \frac{3 \cdot 2! \cdot 4! \cdot 12!}{18!}$$

$$= \frac{3 \cdot 2 \cdot 24 \cdot 12!}{18 \cdot 17 \cdot 16 \cdot 15 \cdot 14 \cdot 13 \cdot 12!}$$

$$= \frac{1}{17 \cdot 2 \cdot 15 \cdot 14 \cdot 13} = \frac{1}{92\,820}$$

Homework: pg. 321 #4, 5, 8*, 10, 11, 12, 14, 16, 18