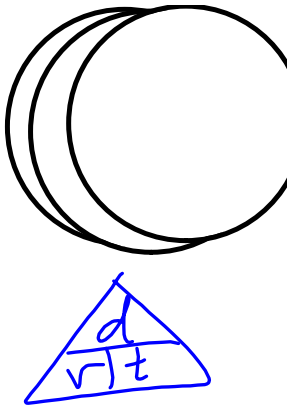
Speed-Distance-Time



$$d = vt \qquad t = \frac{d}{v} \qquad v = \frac{d}{t}$$

$v = \frac{km}{h}$

The Situation:

It takes you a certain amount of time to run a 5000m race. When your friend runs they finish the race 2min before you. When you later analyze the race you realize your friend runs 125m/min faster than you. So how fast do YOU run?

- What don't we know? v = your speed

  friend's speed = $v + 125$

- Find an expression for your time in the race.

  $t = \frac{d}{v} \Rightarrow \boxed{t = \frac{5000}{v}}$

- Find an expression for your friend's time in the race.

  $\boxed{t = \frac{5000}{v+125}}$

- Who was faster? So... which time is smaller? How much smaller?

  friend friend 2 minutes less

- Write that as a "math sentence".

  $t_{you} - t_{friend} = 2 \text{ min}$

  $\boxed{\frac{5000}{v} - \frac{5000}{v+125} = 2}$

How do we solve this?

Multiply both sides by all denominators, simplify & solve.

5.4 Solving Rational Equations

General Strategy: See last page

Example 1: Solve $\frac{2x-6}{x^2-9}=0$ (x^2-9)

$f(x)=\frac{2(x-3)}{(x-3)(x+3)}$

hole at $x-3$; V.A at $x+3$

$2x-6=0$

$2x=6$

$x=3$

but $x^2-9\neq 0$

$(x-3)(x+3)\neq 0$

$x\neq 3,-3$

$\therefore$ no soln

Example 2: Solve $\frac{(x+3)}{x-4}=\frac{(x-1)}{x+2}$ $(x-4)(x+2)$

$x^2+5x+6=x^2-5x+4$

$\frac{10x}{10}=\frac{-2}{10}$

$x=-\frac{1}{5}$ ✓

but $x\neq 4,-2$

Example 3: Solve $\frac{x+8}{(x-4)}=\frac{4x-3}{3}$

$x\neq 4$

$3(x+8)=(x-4)(4x-3)$

$3x+24=4x^2-19x+12$

$0=4x^2-22x-12$

$0=2(2x^2-11x-6)$ (-12)

$0=2(2x+1)(x-6)$

$2x+1=0$

$x=-\frac{1}{2}$ ✓

$x=6$ ✓

or....

$-12\cdot 1=-12$

$-12+1=-11$

$(2x^2-12x)+(x-6)$

$2x(x-6)+1(x-6)$

$=(x-6)(2x+1)$

Example 4: When does the function $f(x)=\dfrac{x^3-4x^2+5x-2}{x^2-9}$ have a value of 3?

$3=\dfrac{x^3-4x^2+5x-2}{(x^2-9)}$ $\quad x\neq\pm3$

$3x^2-27=x^3-4x^2+5x-2$

$0=x^3-7x^2+5x+25$ $\quad$ test $x=\pm1,\pm5,\pm25$

notice that $P(5)=0$ $\therefore (x-5)$ is a factor

5	1	−7	5	25
		5	−10	−25
	1	−2	−5	0

$\Rightarrow 0=(x-5)(x^2-2x-5)$

$x=\dfrac{-(-2)\pm\sqrt{(-2)^2-4(1)(-5)}}{2(1)}$

$=\dfrac{2\pm\sqrt{24}}{2}$

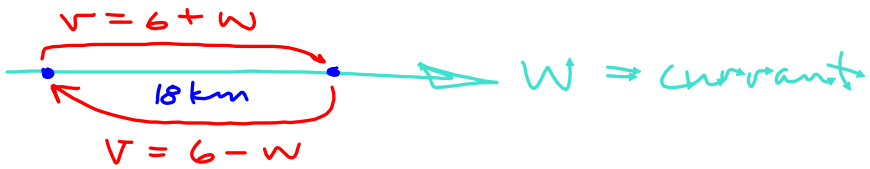
$=1\pm\sqrt{6}$

$=3.445$ or -1.445

$\therefore x=5,\ 3.445$ and -1.445

Example 5: As a fundraiser for Pi Day the enterprising Muffins the Penguin offers a "Tour of the Rhine". He will row a boat full of passengers 18km upstream, stop for lunch and then row back to the starting point. The total trip, including a 2 hour lunch, takes 10 hours. If he can row at a speed of 6km/h, how fast is the current of the river?

$t = \frac{d}{v}$



$t_{Total} \Rightarrow 10 = t_{up} + t_{lunch} + t_{down}$

$8 = t_{up} + t_{down}$

$6 - w \neq 0$, $6 + w \neq 0$, $w \neq 6$

$$\left[8 = \frac{18}{6-w} + \frac{18}{6+w}\right](6-w)(6+w)$$

$$8(6-w)(6+w) = 18(6+w) + 18(6-w)$$

$$8(36-w^2) = 108 + 18w + 108 - 18w$$

$$\frac{8(36-w^2)}{8} = \frac{216}{8}$$

$$36 - w^2 = 27$$

$$\pm\sqrt{9} = \sqrt{w^2}$$

$w = \pm 3$

Good

∴ The river current is 3 km/h

Homefun:

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