5.5 Conditional Probability

Monty Hall dilemma pg. 343

Dependent Events: events whose outcomes are affected by each other

ex. when drawing 2 cards from a deck, the probability of drawing an ace depends on whether or not an ace was drawn the first time

Conditional Probability: the probability of an event occurring given that another event has already occurred

A computer manufacturer knows that, in a box of 100 chips, 3 will be defective. Jocelyn will draw 2 chips, at random, from a box of 100 chips.

? What is the probability that both of the chips will be defective?

EXAMPLE 1 Calculating the probability of two events

$P(\text{def} \mid \text{def})$

Determine the probability that Jocelyn will draw 2 defective chips.

$P(2 \text{ defects}) = \frac{3}{100} \cdot \frac{2}{99} = \frac{6}{9900}$

$= \frac{1}{1650}$

1st defect 2nd defect

Communication Tip

$P(B \mid A)$ is the notation for a conditional probability. It is read "the probability that event B will occur, given that event A has already occurred."

EXAMPLE 2 Calculating the conditional probability of a pair of dependent events

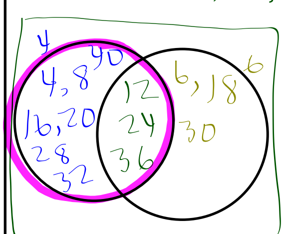
Nathan asks Riel to choose a number between 1 and 40 and then say one fact about the number. Riel says that the number he chose is a multiple of 4. Determine the probability that the number is also a multiple of 6, using each method below.

a) A Venn diagram **b)** A formula

4, 8, 12, 16, 20, 24, 28, 32, 36, 40

$A = \{\text{multiples of } 4\}$

$B = \{\text{multiples of } 6\}$



$P(4 \cap 6) = \frac{3}{10}$

$P(A \text{ and } B) = P(A) \cdot \underbrace{P(B \mid A)}_{\text{what the Q asks}}$

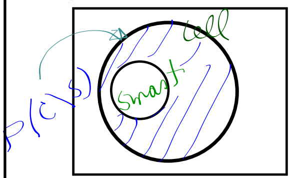
$\frac{P(A \text{ and } B)}{P(A)} = P(B \mid A)$

$\frac{3/40}{10/40} = P(B \mid A)$

$= 3/10$

EXAMPLE 3 Solving a conditional probability problem

According to a survey, 91% of Canadians own a cellphone. Of these people, 42% have a smartphone. Determine, to the nearest percent, the probability that any Canadian you met during the month in which the survey was conducted would have a smartphone.



everyone

$P(S \cap C) = 42\%$ of 91%

$= (42)(.91)$

$= 38.22\%$

$$P(S \mid C) = \frac{P(S \cap C)}{P(C)}$$

$$0.42 = \frac{P(S \cap C)}{0.91}$$

$$(0.42)(0.91) = P(S \cap C)$$

$$0.3822 = P(S \cap C)$$

Your Turn

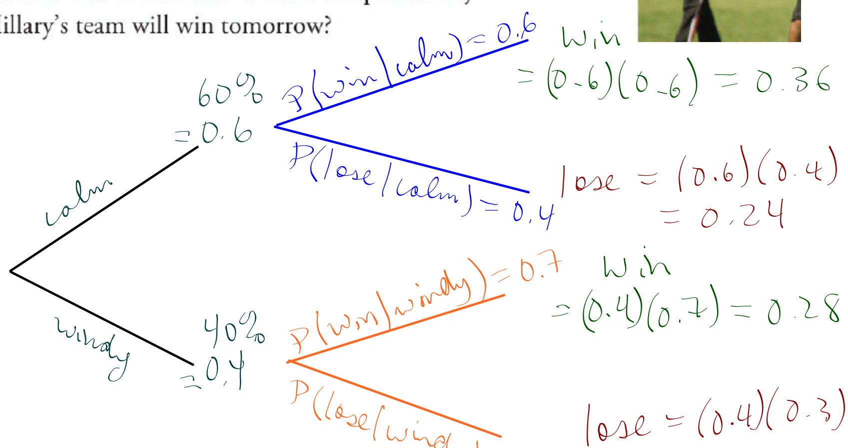
a) Determine, to the nearest percent, the probability that any Canadian you met in that month would have a cellphone but not a smartphone.

b) How could you represent this probability in a Venn diagram?

a) $91\% - 38.22\% = 52.78\%$

EXAMPLE 4 Making predictions that involve dependent events

Hillary is the coach of a junior ultimate team. Based on the team's record, it has a 60% chance of winning on calm days and a 70% chance of winning on windy days. Tomorrow, there is a 40% chance of high winds. There are no ties in ultimate. What is the probability that Hillary's team will win tomorrow?



Practice: pg. 350 # 4, 5, 7, 9*, 11, 13, 16, 18, 21

$P(\text{win}) = P(\text{win}|\text{windy}) + P(\text{win}|\text{calm})$

$= .28 + 0.36$

$= 0.64 \Rightarrow$ they have a 64% chance of winning

$P(\text{lose}) = 0.12 + 0.24$

$= 36\%$

$P(\text{win}) + P(\text{lose})$

$= 64\% + 36\%$

$= 100\% \leftarrow$ no ties

In Summary

Key Ideas

- If the probability of one event depends on the probability of another event, then these events are called **dependent events**. For example, drawing a heart from a standard deck of 52 playing cards and then drawing another heart from the same deck without replacing the first card are dependent events.
- If event B depends on event A occurring, then the **conditional probability** that event B will occur, given that event A has occurred, can be represented as follows:

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)}$$

Need to Know

- If event B depends on event A occurring, then the probability that both events will occur can be represented as follows:

$$P(A \cap B) = P(A) \cdot P(B \mid A)$$

- A tree diagram is often useful for modelling problems that involve dependent events.
- Drawing an item and then drawing another item, without replacing the first item, results in a pair of dependent events.