5.5 Solving Rational Function Inequalities

General Strategy: - everything to one side

Example 1: Solve $\dfrac{240}{a+8} > \dfrac{20a}{a+1}$

$a \neq -1, -8$

$$\left(\frac{240}{a+8} - \frac{20a}{a+1} > 0\right)(a+8)(a+1)$$

Since I don't know if either (a+8) or (a+1) is ⊕ or ⊖, change to an equal = AND SOLVE!

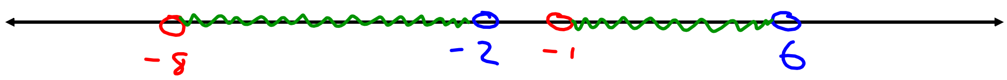
$$240(a+1) - 20a(a+8) \cancel{>} 0$$
$$240a + 240 - 20a^2 - 160a = 0$$
$$-20a^2 + 80a + 240 = 0$$
$$-20(a^2 - 4a - 12) = 0$$
$$-20(a-6)(a+2) = 0$$
$$\boxed{a = 6} \quad \boxed{a = -2}$$

points of intersection

So test around these but... $\boxed{a \neq -8, -1}$ are V.A ... so test around these too!

always hollow

$\therefore$ test around $\boxed{x = -8, -2, -1, 6}$



test $a = -10$: $\dfrac{240}{-10+8} > \dfrac{20(-10)}{-10+1}$

in original eqn

$-120 > \dfrac{-200}{-9}$ false! $\therefore$ the soln is not left of -8

test $a = -5$: $\dfrac{240}{3} > \dfrac{-100}{-4}$ true ☺ $\therefore$ the soln interval contains values on $a \in (-8, -2)$ ☆

test $a = -1.5$: false

test $a = 0$: true

test $a = 10$: false

$\therefore$ soln $(-8, -2) \cup (-1, 6)$

Example 2: Solve $\dfrac{x+3}{x+1} \ge \dfrac{x-2}{x-3}$

$$\boxed{x \neq -1, 3}$$

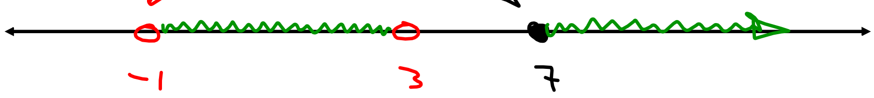
$$(x+3)(x-3) = (x-2)(x+1)$$

$$(\cancel{x^2} - 9) = \cancel{x^2} - x - 2$$

$\boxed{x = 7}$ $\Leftarrow$ since this is a POI, we must consider the inequality

$\geq$ yields a closed boundary.

open boundaries caused by V.A.



test $x = -2$: $\dfrac{-2+3}{-2+1} \geq \dfrac{-2-2}{-2-3}$

$\dfrac{1}{-1} \geq \dfrac{-4}{-5}$ false!

test $x = 0$: $\dfrac{3}{1} \geq \dfrac{-2}{-3}$ true!

test $x = 4$: $\dfrac{7}{5} \geq \dfrac{2}{1}$ false

test $x = 8$: $\dfrac{11}{9} \geq \dfrac{6}{5}$ true

$$\therefore \; x \in (-1, 3) \cup [7, \infty)$$

Note round brackets for V.A.

Example 3: The total resistance in a parallel circuit can be determined using the equation:

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}$$

(a) if the resistance of one resistor is x and the other is x + 10 determine an expression for the total resistance of the circuit

(b) find out for what values of x the total resistance will be greater than 6

a) $\frac{1}{R_T} = \frac{1}{x} + \frac{1}{x+10}$

$\frac{1}{R_T} = \frac{x+10}{x(x+10)} + \frac{x}{x(x+10)}$

$\left[\frac{1}{R_T} = \frac{2x+10}{x(x+10)}\right]^{-1}$

$R_T = \frac{x(x+10)}{2(x+5)}$

$\boxed{x \neq -5}$

b) $6 < \frac{x(x+10)}{2(x+5)}$

$12(x+5) = x(x+10)$

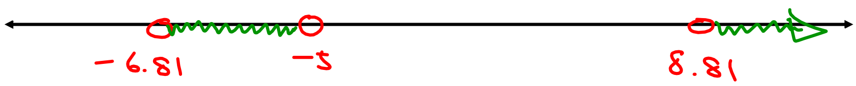
$12x + 60 = x^2 + 10x$

$0 = x^2 - 2x - 60$

Quadratic form.

$x = 1 + \sqrt{61} \doteq 8.81$

$x = 1 - \sqrt{61} \doteq -6.81$



test $x = -7$: $6 < 5.25$ false

$x = -6$: $6 < 12$ ✓ true

$x = 0$: $6 < 0$ ✗

$x = 9$: $6 < 6.11$ ✓ true

Since resistance must be positive $\therefore$

$R_T > 6$ for $x \in (1+\sqrt{61}, \infty)$

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