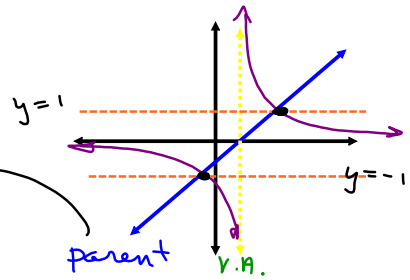
Review: Rational Functions



Day 1: Reciprocal Functions $y = \frac{1}{f(x)}$

$f(x)$	reciprocal
Zeroes →	V.A.
y-int(s) →	$\frac{1}{\text{y-int.}}$ $(\frac{1}{5})$
domain $(x \in \mathbb{R})$ →	$x \neq$ V.A.
E.B. $y \to \pm\infty$ →	$y \to 0$ (from above or below)

↳ always gives a H.A. of $y = 0$

where parent crosses x-axis

* invariant points occur @ $y = \pm 1$

Day 2 and 3: Graphing General Rational Functions

$$f(x) = \frac{P(x)}{Q(x)}$$

$P(x) = 0$ → Zeroes of the top are the zeroes of $f(x)$

$Q(x) = 0$ → Zeroes of bottom yield V.A. or "holes" of $f(x)$

* when factors of $P(x)$ cancel with factors of $Q(x)$ → $f(x) = \frac{x-3}{(x+3)(x-3)}$

hole @ $x = 3$

behaves like $f(x) = \frac{1}{x+3}$

$f(3) = \frac{1}{3+3}$

∴ hole @ $(3, \frac{1}{6})$

End Behaviour

↳ H.A. of $y = 0$... if top degree > bottom degree

↳ H.A of $y = \#$... if top degree = bottom degree

ex// $y = \frac{8x-3}{2x+1} \Rightarrow$ H.A. $y = \frac{8x}{2x}$

$\boxed{y = 4}$

↳ O.A. of $y = g(x)$ ... if top degree > bottom degree by 1

ex// $R(x) = \frac{3x^2 + 2x - 1}{2x + 5}$

$\Rightarrow$ O.A of $y = \frac{3x^2}{2x}$

$\boxed{y = \frac{3x}{2}}$

* may also have parabolic asymptotes!

ex// $R(x) = \frac{4x^3}{\sim} \Rightarrow$ P.A. of $y = 4x^2$

Day 4: Solving Rational Functions

$$\frac{x+3}{-4x} = \frac{x-1}{-4}$$

$-4x \neq 0$

$x \neq 0$

$-4(x+3) = -4x(x-1)$

$-4x - 12 = -4x^2 + 4x$

$4x^2 - 8x - 12 = 0$

$4(x^2 - 2x - 3) = 0$

$4(x-3)(x+1) = 0$

$x = 3, -1$

Day 5: Solving Rational Inequalities

$$\frac{x}{6x-9} \leq \frac{1}{x}$$

$6x - 9 \neq 0$

$6x \neq 9$

$x \neq 3/2$

$x \neq 0$

$x^2 = 6x - 9$

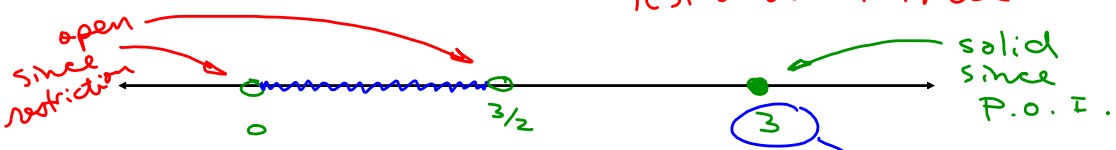
$x^2 - 6x + 9 = 0$

$(x-3)(x-3) = 0$

zeroes @ $x = 3$

key points @ $x = 0, 3/2, 3$

test around these



test $x = -1 \Rightarrow \frac{-1}{-6-9} \leq \frac{1}{-1}$ false

test $x = 1 \Rightarrow \frac{1}{6-9} \leq \frac{1}{1}$ true

test $x = 2 \Rightarrow \frac{2}{6(2)-9} \leq \frac{1}{2}$ false

test $x = 4 \Rightarrow \frac{4}{24-9} \leq \frac{1}{4}$

$\frac{4}{15} \leq \frac{1}{4}$ false

$\therefore$ Soln interval is $x \in (0, 3/2) \cup x = 3$

Day 6: AROC and IROC of RF's

$$\text{AROC} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

$$\text{IROC} \Rightarrow DQ = \frac{f(a+h) - f(a)}{h}$$

for very small h

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