

6.1 Rational Expressions

Definition: A rational expression is an algebraic expression whereby the numerator **AND** denominator are polynomials.

ex. $\frac{4x^2+2x-5}{1}$ or $\frac{3}{x+1}$ or $\frac{x^2-1}{x+3}$ NOT $\frac{\sqrt{2x-1}}{3x}$

* When working with rational expressions, we must consider the restrictions on the domain. These come from cases when we try to **divide by zero**.

ex. $\frac{5x+1}{3x+4}$

restrictions

$$3x+4 \neq 0$$

$$3x \neq -4$$

$$x \neq -\frac{4}{3}$$

$\therefore -\frac{4}{3}$ is non-permissible

$$\text{Domain: } \{x \in \mathbb{R} \mid x \neq -\frac{4}{3}\}$$

ex. $\frac{3}{(x+1)(x-2)}$

$$x+1 \neq 0$$

$$x \neq -1$$

$$x-2 \neq 0$$

$$x \neq 2$$

$$D: \{x \in \mathbb{R} \mid x \neq -1, 2\}$$

ex. $\frac{2x-1}{x^2-x-12}$

$$x^2-x-12 \neq 0$$

$$(x+3)(x-4) \neq 0$$

$$x+3=0 \quad x-4 \neq 0$$

$$x \neq -3$$

$$x \neq 4$$

$$D: \{x \in \mathbb{R} \mid x \neq -3, 4\}$$

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ex // $\frac{2y^2}{y^2-4}$

$$y^2-4 \neq 0$$

$$(y-2)(y+2) \neq 0$$

$$y \neq \pm 2$$

Definition: Two rational expressions are **equivalent** if we multiply or divide both the **numerator AND denominator** by the same non-zero expression.

ex. $\frac{3(x+1)}{(x-2)(x+1)}$ $\xrightarrow{\text{equivalent}}$ ex. $\frac{3}{(x-2)} \cdot \frac{(x+1)}{x+1}$

$\frac{3x+3}{x^2-x-2}$ $\xrightarrow{\text{still equivalent}}$ $\frac{3}{x-2}$

NOTE: two equivalent rational expressions do not necessarily have the same domain restrictions!!!!

ex. $\frac{3(x+1)}{(x-2)(x+1)}$
 $x \neq 2$ and $x \neq -1$

ex. $\frac{3}{(x-2)}$
 $x \neq 2$

restrictions must be stated BEFORE simplifying rational expressions!!

Method: to simplify a rational expression...

② State restrictions

① Factor the numerator and the denominator and cancel anything they have in **common**.

ex. $\frac{16x^2 - 9y^2}{8x - 6y}$

$4x - 3y \neq 0$
 $4x \neq 3y$
 $x \neq \frac{3}{4}y$

$= \frac{(4x+3y)(4x-3y)}{2(4x-3y)}$
 $= \frac{4x+3y}{2}$

ex. $\frac{6x^2 + 7x - 5}{3x^2 - 7x - 20}$

$3x+5 \neq 0$ $x-4 \neq 0$
 $x \neq -\frac{5}{3}$ $x \neq 4$
 $10 \cdot -3 = -30$
 $10 + -3 = 7$

$= \frac{(3x+5)(2x-1)}{(3x+5)(x-4)} = \frac{2x-1}{x-4}$
 $= (6x^2 + 10x) \div (-3x - 5)$
 $= 2x(3x+5) \div (3x+5)$
 $= (3x+5)(2x-1)$