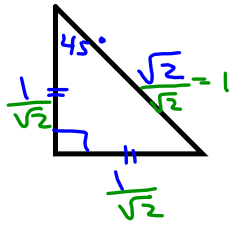
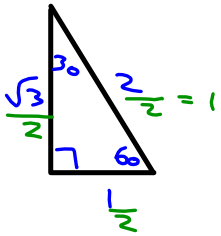
6.2 Angles on the Cartesian Plane

Special Angles in Radians

degrees	0	30	45	60	90
radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
sinA	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cosA	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tanA	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∅

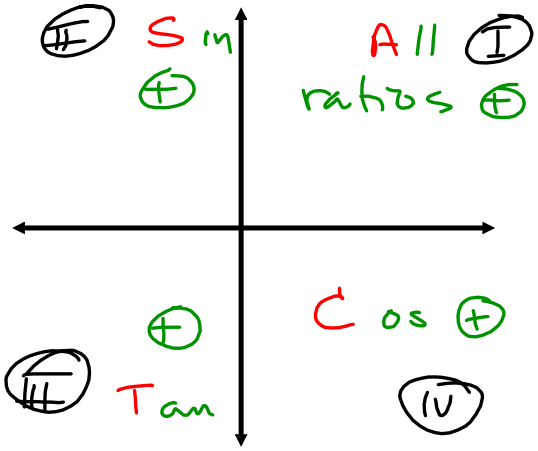
$\frac{\sin}{\cos}$



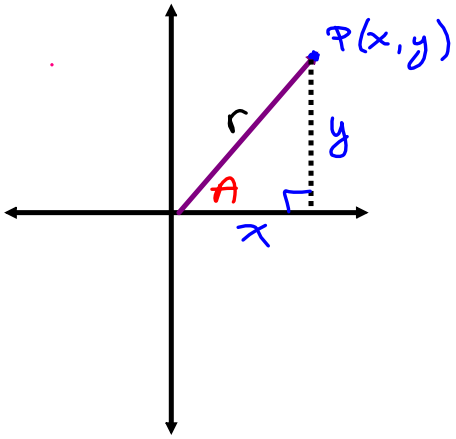


$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

The CAST Rule (or the ASTC rule as I like to call it)



Why?



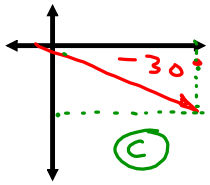
Then sin A = y/r

cos A = x/r

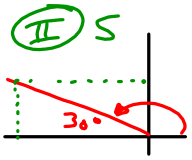
tan A = y/x

Combining the Special Angles with the CAST Rule:

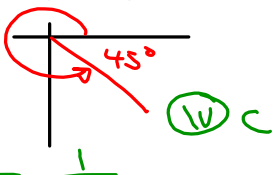
Example 1: Determine the exact value of...

$\cos\left(-\frac{\pi}{6}\right)$ 

$= \frac{\sqrt{3}}{2}$

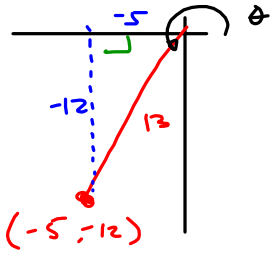
$\sin\left(\frac{5\pi}{6}\right)$ 

$= \frac{1}{2}$

$\cos\left(\frac{7\pi}{4}\right)$ 

$= \frac{1}{\sqrt{2}}$

Example 2: Write the primary trig ratios for the principal angle formed by the point (-5, -12)



$r^2 = (-5)^2 + (-12)^2$
$= 25 + 144$
$\sqrt{r^2} = \sqrt{169}$
$r = 13$

$\sin\theta = -\frac{12}{13}$

$\cos\theta = -\frac{5}{13}$

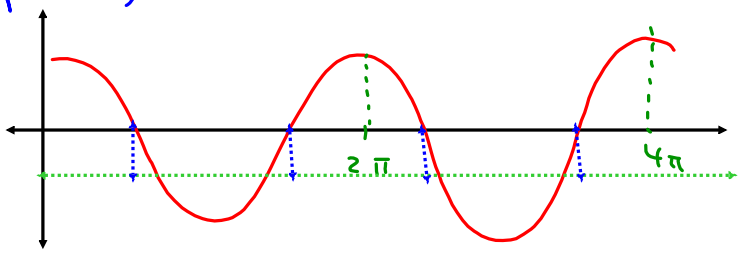
$\tan\theta = \frac{12}{5}$

angle in radians means answer in radians

Example 3: On the interval $[0, 4\pi]$ find the solutions to

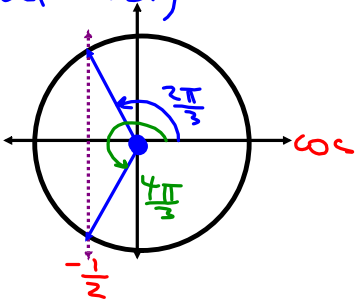
graphically

$\cos A = -0.5$



I should have 4 answers!

unit circly



$\therefore A = \frac{2\pi}{3}$ and $\frac{4\pi}{3}$ $\quad 0 \le A \le 2\pi$

and $\frac{2\pi}{3} + 2\pi = \frac{8\pi}{3}$

and $\frac{4\pi}{3} + \frac{6\pi}{3} = \frac{10\pi}{3}$

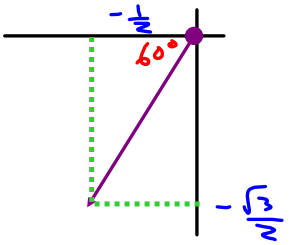
$\therefore A = \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3}$

Reciprocal Trigonometric Ratios

Definition:

csc A = $\frac{1}{\sin A}$ sec A = $\frac{1}{\cos A}$ cot A = $\frac{1}{\tan A} = \frac{\cos A}{\sin A}$

Example 4: Calculate the exact value of $\sec\left(\frac{4\pi}{3}\right) = \frac{1}{\cos\frac{4\pi}{3}}$



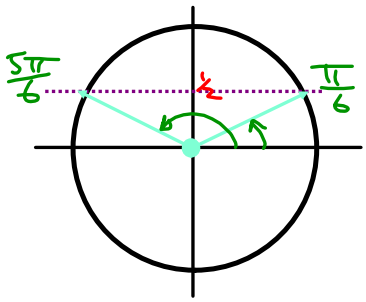
$\cos\frac{4\pi}{3} = \frac{-1}{2}$

$\therefore \sec\frac{4\pi}{3} = \frac{1}{-\frac{1}{2}} = -2$

Example 5: Solve $\csc A = 2$ on the interval $[0, 2\pi]$

this also means $\frac{1}{\sin A} = 2$

$\frac{1}{2} = \sin A$



$A = \frac{\pi}{6}, \frac{5\pi}{6}$

on $[0, 2\pi]$

Example 6: Given the equation below, find the angle to the nearest hundredth.

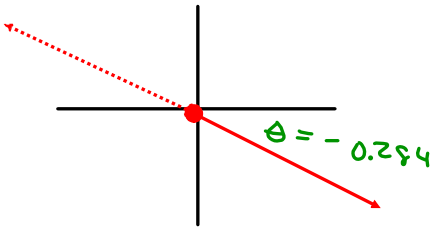
$$\cot\theta = -\frac{24}{7}$$

$\left[\frac{1}{\tan\theta} = -\frac{24}{7}\right]^{-1}$

$\theta = \tan^{-1}\left(-\frac{7}{24}\right)$

$\theta \doteq -0.284$ ← not in the interval

...so add π and 2π



$\theta = -.284 + \pi \doteq 2.86$

$\theta = -.284 + 2\pi \doteq 6.00$

Homefun:

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