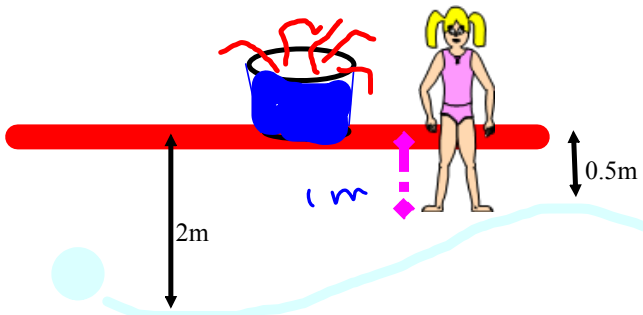
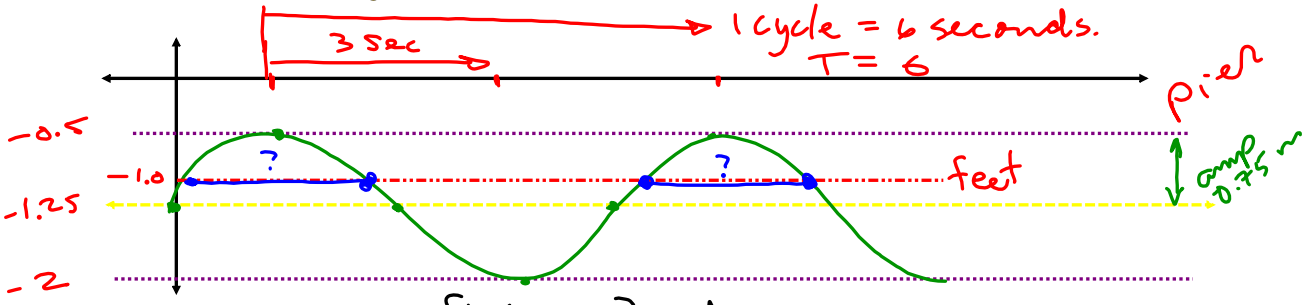
6.6 Modelling with Trig Functions



Pauline sits on the edge of a pier feeding worms to the fishies. The local fisherman are totally choked, however, Pauline is oblivious and only wants to cool off her feet while feeding the fish. If she stretches, she can hang her feet 1 m off the edge of the peer. The time between a low and high point on the waves is 3 seconds. For how long are Pauline's feet in the water each time a wave passes?

let's start with $y = \sin(t)$



$y = a\sin[k(x-c)] + d$

$a = 0.75 \quad c = 0 \quad d = -1.25$

Now find k: $T = \frac{2\pi}{k} \Rightarrow k = \frac{2\pi}{T} = \frac{2\pi}{6} = \frac{\pi}{3}$

$\therefore \boxed{h(t) = 0.75\sin\frac{\pi}{3}t - 1.25}$

? what are we looking for? ... when $h(t) = -1.0$

graph $h(t)$ & $y = -1$ & find POI

1st POI @ $t = 0.3245$

2nd POI @ $t = 2.6755$

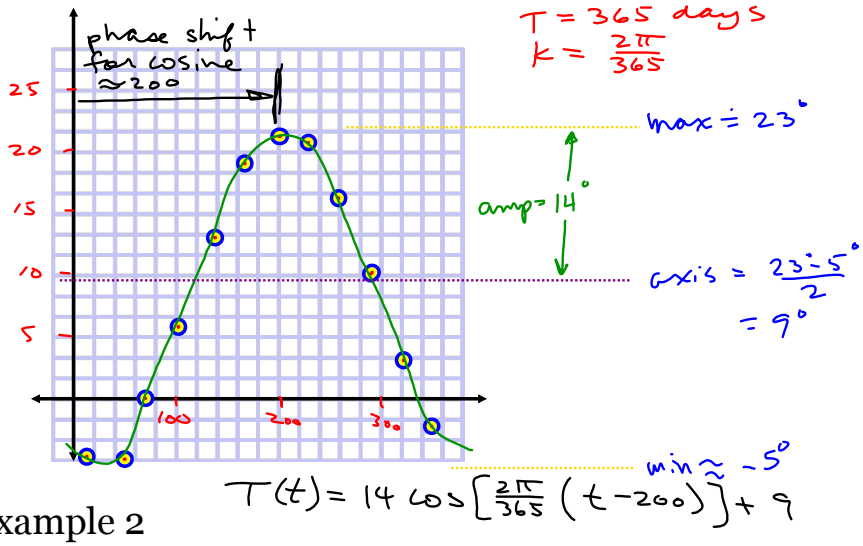
∴ Pauline's feet are in the water for

$2.6755 - 0.3245 \doteq 2.35$ seconds

The following table shows the average monthly means of the daily (24 h) temperatures in Hamilton, Ontario. Each month's average temperature is represented by the day in the middle of the month.

Month	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sep.	Oct.	Nov.	Dec.
Day of Year	15	45	75	106	136	167	197	228	259	289	320	350
°C	−4.8	−4.8	−0.2	6.6	12.7	18.6	21.9	20.7	16.4	10.5	3.6	−2.3

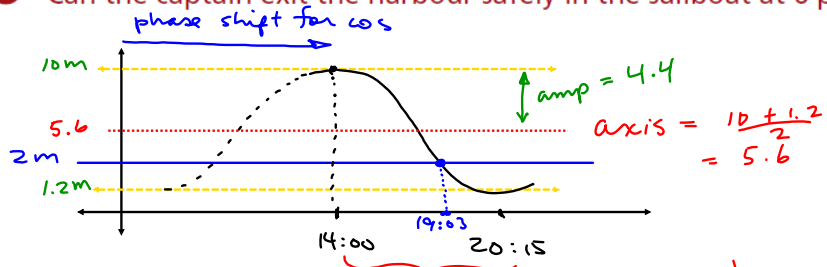
a) Plot the temperature data for Hamilton, and fit a sinusoidal curve to the points.
b) Estimate the average daily temperature in Hamilton on the 200th day of the year.



Example 2

The tides at Cape Capstan, New Brunswick, change the depth of the water in the harbour. On one day in October, the tides have a high point of approximately 10 m at 2 p.m. and a low point of approximately 1.2 m at 8:15 p.m. A particular sailboat has a *draft* of 2 m. This means it can only move in water that is at least 2 m deep. The captain of the sailboat plans to exit the harbour at 6:30 p.m.

? Can the captain exit the harbour safely in the sailboat at 6 p.m.?



$h(t) = 4.4 \cos\left[\frac{4\pi}{25}(t-14)\right] + 5.6$ $\quad k = \frac{2\pi}{12.5} = \frac{4\pi}{25}$

graph and notice that $h(t) = 2$ m @ $t = 19:03$ ... what does that mean?

the water level is high enough until 7 pm so he is safe to leave @ 6 pm.

Example 3

The population size, O, of owls (predators) in a certain region can be modelled by the function $O(t) = 1000 + 100 \sin\left(\frac{\pi t}{12}\right)$, where t represents the time in months and $t = 0$ represents January. The population size, m, of mice (prey) in the same region is given by the function $m(t) = 20\,000 + 4000 \cos\left(\frac{\pi t}{12}\right)$.

a) Sketch the graphs of these functions.
b) Compare the graphs, and discuss the relationships between the two populations.
c) How does the mice-to-owls ratio change over time?
d) When is there the most food per owl? When is it safest for the mice?

Recall: axis =

amplitude =

period =

phase shift =

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