7.1 Exploring Exponential Functions

variable exponent

exponential function: a function of the form $y = a(b)^x$ where $a \neq 0, b > 0, b \neq 1$

creates a discontinuous function

Getting started pg. 434

How are the number of folds and the number of layers of paper related?

A. Copy and complete the second column of this table.

Number of Folds	Number of Layers Created	
0	1	$= 2^0$
1	2	$= 2^1$
2	4	$= 2^2$
3	8	$= 2^3$
4	16	

B. $4 = 2^2$
$\times 16 = 2^4$
$64 = 2^6$

when we multiply same bases, we add the exponents

B. Choose any two numbers under "Number of Layers Created." Determine the product of the two numbers. Write the two numbers and their product as powers of 2.

C. Repeat part B two more times with a different pair of numbers.

D. Is there a pattern that relates the exponents of the powers you multiplied to the exponent of the product? Explain. $x^a \cdot x^b = x^{a+b}$

E. What exponent law does your answer to part D show?

$2^5 = 32$
$\div 2^2 = 4$
$2^3 = 8$

$2^{5-2} = 2^3$

F. In the blank column in the table, summarize your findings using powers of 2. Label the column "Number of Layers as a Power of 2."

G. Choose any two numbers under "Number of Layers Created." Divide the greater number by the lesser number. Write the two numbers and the quotient as powers of 2. Repeat with a different pair of numbers.

H. Examine the exponents of the numbers you wrote in part G. What exponent law do they show? when we divide same bases, we subtract their exponents

I. Explain how you can predict the number of layers created given the number of folds made. # layers $= 2^{(\# \text{ of folds})}$

J. Suppose that you continued to fold the sheet of paper and count the number of layers you created. Would the pattern you observed continue forever? Explain. → hypothetically → reality has limits

Consider the following exponential functions and their graphs...

$a = 1$ $\qquad$ $a = 2$ $\qquad$ $a = 1$ $\qquad$ $a = 8$

$f(x) = 10^x$ $\qquad$ $g(x) = 2(5)^x$ $\qquad$ $h(x) = \left(\frac{1}{2}\right)^x = 2^{-x}$ $\qquad$ $j(x) = 8\left(\frac{1}{4}\right)^x$

x	$f(x)$
−3	$10^{-3} = 0.001$
−2	$10^{-2} = 0.01$
−1	$10^{-1} = 0.1$
0	$10^0 = 1$
1	$10^1 = 10$
2	$10^2 = 100$
3	$10^3 = 1000$

$\div 10$ $\quad \div 10$ $\quad \times 10$ $\quad \times 10$ $\quad \times 10$

x	$g(x)$
−3	$2(5)^{-3} = 0.016$
−2	$2(5)^{-2} = 0.08$
−1	$2(5)^{-1} = 0.4$
0	$2(5)^0 = 2$
1	$2(5)^1 = 10$
2	$2(5)^2 = 50$
3	$2(5)^3 = 250$

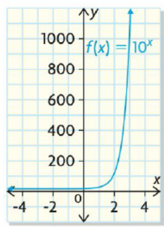
$\times 5$

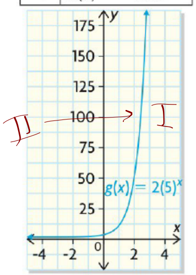
x	$h(x)$
−3	$\left(\frac{1}{2}\right)^{-3} = 8$
−2	$\left(\frac{1}{2}\right)^{-2} = 4$
−1	$\left(\frac{1}{2}\right)^{-1} = 2$
0	$\left(\frac{1}{2}\right)^{0} = 1$
1	$\left(\frac{1}{2}\right)^{1} = 0.5$
2	$\left(\frac{1}{2}\right)^{2} = 0.25$
3	$\left(\frac{1}{2}\right)^{3} = 0.125$

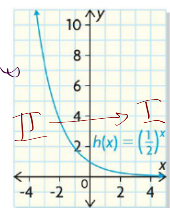
$\times \frac{1}{2}$

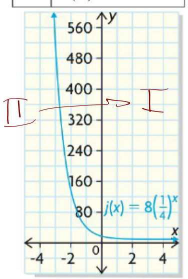
x	$j(x)$
−3	$8\left(\frac{1}{4}\right)^{-3} = 512$
−2	$8\left(\frac{1}{4}\right)^{-2} = 128$
−1	$8\left(\frac{1}{4}\right)^{-1} = 32$
0	$8\left(\frac{1}{4}\right)^{0} = 8$
1	$8\left(\frac{1}{4}\right)^{1} = 2$
2	$8\left(\frac{1}{4}\right)^{2} = 0.5$
3	$8\left(\frac{1}{4}\right)^{3} = 0.125$

$\times \frac{1}{4}$









* all exponential functions of this type have the line $y = 0$ (the x-axis) as an asymptote

Reflecting

A. Based on your observations, which characteristics of the graphs of exponential functions are similar to characteristics of polynomial functions you have studied and which are different?

exponents are natural #s

exponents are variables

$x \in \mathbb{R}$ for both

B. Kent claims that all exponential functions of the form $y = a(b)^x$, where $a > 0$, $b > 0$, and $b \neq 1$, do not have x-intercepts. Do you agree or disagree? Explain. agree... it is impossible for $a(b)^x < 0$ with these restrictions

C. Francine claims that all exponential functions of this form have a restricted range. Do you agree or disagree? Explain. $y > 0$

D. Can you use the end behaviour of exponential functions of this form to help you decide if the function is increasing or decreasing? Explain. nope... both go from II → I

E. Describe any patterns you noticed in the tables of values for the exponential functions you investigated.

a = y-int.

b = growth or decay rate

Key Ideas

- An exponential function has the form $f(x) = a(b)^x$, where x is the exponent and $a \neq 0$, $b > 0$, and $b \neq 1$.
- All exponential functions of the form $f(x) = a(b)^x$, where $a > 0$, $b > 0$, and $b \neq 1$, have the following characteristics:

Number of x-Intercepts	0
y-Intercept	a
End Behaviour	Curve extends from quadrant II to quadrant I.
Domain	$\{x \mid x \in \mathrm{R}\}$
Range	$\{y \mid y > 0, y \in \mathrm{R}\}$

Homefun: Pg. 439 # 1-3
+ Read Need to Know

increasing if $b > 1$, decreasing if $0 < b < 1$