7.2 Compound Angle Formulas

The Addition and Subtraction Identities for Sin and Cos:

$$\boxed{\begin{aligned}\sin(A+B)&=\sin A\cos B+\cos A\sin B\\ \sin(A-B)&=\sin A\cos B-\cos A\sin B\\ \cos(A+B)&=\cos A\cos B-\sin A\sin B\\ \cos(A-B)&=\cos A\cos B+\sin A\sin B\end{aligned}}$$

$\frac{5\pi}{12}=75^\circ$
$=30^\circ+45^\circ$
$=\frac{\pi}{6}+\frac{\pi}{4}$

$\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2}$

Example 1: Find the exact values of sin(5π/12) and cos(5π/12)

$\sin\left(\frac{5\pi}{12}\right)=\sin\left(\frac{\pi}{6}+\frac{\pi}{4}\right)$
$=\sin\frac{\pi}{6}\cos\frac{\pi}{4}+\cos\frac{\pi}{6}\sin\frac{\pi}{4}$
$=\left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)+\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$
$=\frac{\sqrt{2}}{4}+\frac{\sqrt{6}}{4}$
$=\frac{\sqrt{2}+\sqrt{6}}{4}$

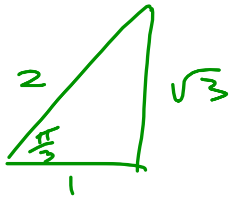
$\cos\left(\frac{\pi}{6}+\frac{\pi}{4}\right)$
$=\cos\frac{\pi}{6}\cos\frac{\pi}{4}-\sin\frac{\pi}{6}\sin\frac{\pi}{4}$
$=\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)-\left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$
$=\frac{\sqrt{6}}{4}-\frac{\sqrt{2}}{4}=\frac{\sqrt{6}-\sqrt{2}}{4}$

Example 2: Rewrite the following in terms of sin(x) and cos(x) $\sec\left(\frac{\pi}{2}+x\right)$

$=\frac{1}{\cos\left(\frac{\pi}{2}+x\right)}$
$=\frac{1}{\underbrace{\cos\frac{\pi}{2}}_{0}\cos x-\underbrace{\sin\frac{\pi}{2}}_{1}\sin x}$
$=\frac{1}{0-\sin x}$
$=\frac{1}{-\sin x}=-\csc x$

Addition and Subtraction Angle Identities for Tangent:

$$\tan(A+B) \equiv \frac{\tan A + \tan B}{1 - \tan A \tan B} \qquad \tan(A-B) \equiv \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

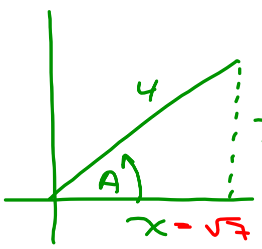


Ex 3: Use an identity to write an equivalent expression for: $\tan\left(\frac{7\pi}{6} - \frac{\pi}{3}\right)$

$$= \frac{\tan\left(\frac{7\pi}{6}\right) - \tan\left(\frac{\pi}{3}\right)}{1 + \tan\left(\frac{7\pi}{6}\right)\tan\left(\frac{\pi}{3}\right)}$$

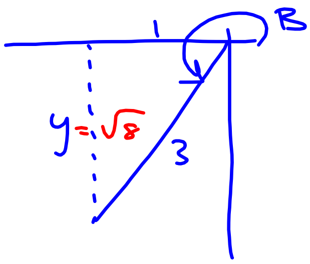
$$= \frac{\left(\frac{1}{\sqrt{3}}\right) - \sqrt{3}\cdot\frac{\sqrt{3}}{\sqrt{3}}}{1 + \left(\frac{1}{\sqrt{3}}\right)\left(\sqrt{3}\right)} = \frac{\frac{1}{\sqrt{3}} - \frac{3}{\sqrt{3}}}{2} = \frac{\frac{-2}{\sqrt{3}}}{2} = \frac{-2}{\sqrt{3}}\cdot\frac{1}{2} = -\frac{1}{\sqrt{3}}$$

Example 4. Find an exact value for sin(A + B) if A is a QI angle, B is a QIII angle, and sin(A) = 3/4 and cos(B) = -1/3.



$x = \sqrt{4^2 - 3^2}$

$x = \sqrt{7}$



$y = \sqrt{3^2 - 1^2}$

$= \sqrt{8}$

$\sin(A+B)$

$= \sin A \cos B + \cos A \sin B$

$= \left(\frac{3}{4}\right)\left(-\frac{1}{3}\right) + \left(\frac{\sqrt{7}}{4}\right)\left(\frac{\sqrt{8}}{3}\right)$

$= \frac{-3}{12} - \frac{\sqrt{56}}{12}$

$= \frac{-3 - \sqrt{56}}{12}$

Example 5: Rewrite $\cos(\pi/2 - x)$

$= \cancel{\cos\frac{\pi}{2}}^{\,0} \cos x + \cancel{\sin\frac{\pi}{2}}^{\,1} \sin x$

$= \sin x$

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