The Challenge:

Develop three new formulas

The DOUBLE ANGLE FORMULAS

i.e. sin 2x = $2\sin x \cos x$

cos 2x =

tan 2x =

Do this by using yesterday's formulas and the fact that

sin 2x = sin (x + x) = ... you do the rest...

$= \sin x \cos x + \cos x \sin x$

$= 2\sin x \cos x$

$\boxed{\cos 2x} = \cos(x+x)$

$= \cos x \cos x - \sin x \sin x$

$= \boxed{\cos^2 x - \sin^2 x}$

$\boxed{\sin^2 x + \cos^2 x = 1}$

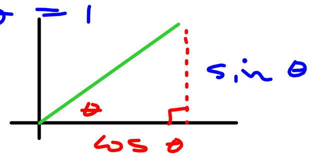
$= (1-\sin^2 x) - \sin^2 x$

$= \boxed{1 - 2\sin^2 x}$

or $\cos^2 x - (1 - \cos^2 x)$

$= \boxed{2\cos^2 x - 1}$

$\sin^2\theta + \cos^2\theta = 1$



7.3 THE DOUBLE ANGLE FORMULAS

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

Other forms...

$$= 1 - 2\sin^2\theta$$

$$= 2\cos^2\theta - 1$$

Example 1: simplify the following expression and then evaluate.

$$2\sin\frac{\pi}{8}\cos\frac{\pi}{8}$$

looks like

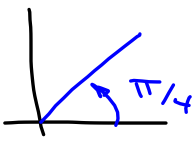
$$2\sin\theta\cos\theta = \sin 2\theta$$

where $\theta = \frac{\pi}{8}$

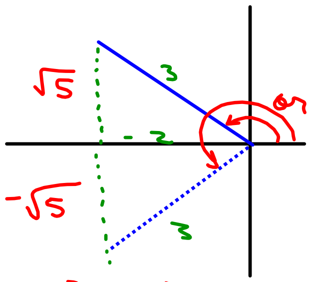
$$\sin 2\left(\frac{\pi}{8}\right)$$

$$= \sin\frac{\pi}{4}$$

$$= \frac{\sqrt{2}}{2} \text{ or } \frac{1}{\sqrt{2}}$$



Example 2: If cosA = -2/3, determine possible values of cos2A and sin2A



$y^2 = 3^2 - (-2)^2$

$y = \sqrt{9-4}$

$y = \sqrt{5}$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= \left(-\frac{2}{3}\right)^2 - \left(\pm\frac{\sqrt{5}}{3}\right)^2$$

$$= \frac{4}{9} - \frac{5}{9}$$

$$= -1/9$$

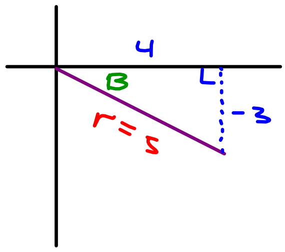
regardless of whether A is in QII or QIII

$$\sin 2A = 2\sin A\cos A$$

$$= 2\left(\frac{\pm\sqrt{5}}{3}\right)\left(-\frac{2}{3}\right)$$

$$= \frac{\pm 4\sqrt{5}}{9}$$

Example 3: If tanB = -3/4, and B is a QIV angle, determine the value of cos2B



$$3^2 + 4^2 = 5^2$$
$$5^2 + 12^2 = 13^2$$

$$\cos 2B = \cos^2 B - \sin^2 B$$
$$= \left(\tfrac{4}{5}\right)^2 - \left(\tfrac{-3}{5}\right)^2$$
$$= \tfrac{16}{25} - \tfrac{9}{25}$$
$$= \tfrac{7}{25}$$

Example 4: Develop a formula for sin(x/2)... THE HALF ANGLE FORMULA

$$\frac{1}{2}\boxed{\sin 2\theta = 2\sin\theta\cos\theta}\frac{}{2}$$

think ...

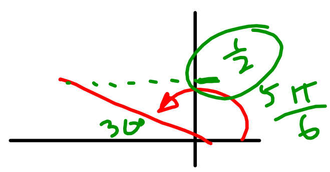
$$\sin\left(\tfrac{A}{2} + \tfrac{A}{2}\right)$$

$$\sin A = 2\sin\left(\tfrac{A}{2} + \tfrac{A}{2}\right)$$

ex// $\sin\left(\frac{5\pi}{3}\right) = 2\sin\left(\frac{5\pi}{6}\right)\cos\left(\frac{5\pi}{6}\right)$ ($\div 2$)

ex// $\sin\left(\frac{5\pi}{12}\right)\cos\left(\frac{5\pi}{12}\right) \Rightarrow$ solve exactly

$$\tfrac{1}{2}\sin\left(2\cdot\tfrac{5\pi}{12}\right) = \tfrac{2}{2}\sin\left(\tfrac{5\pi}{12}\right)\cos\left(\tfrac{5\pi}{12}\right)$$
$$= \tfrac{1}{2}\sin\left(\tfrac{5\pi}{6}\right) = \tfrac{1}{2}\left(\tfrac{1}{2}\right)$$
$$= \tfrac{1}{4}$$



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