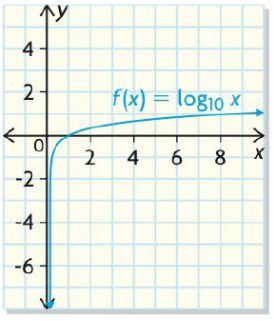
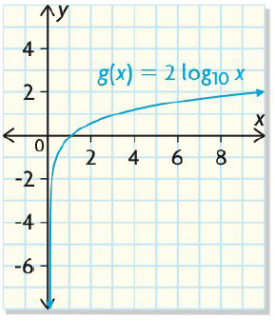
7.4 Characteristics of Logarithmic Functions with Base 10 and Base *e*

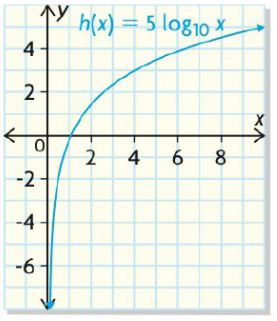
x	$f(x) = \log_{10} x$
−1	undefined
0	undefined
1	0
2	0.301...
3	0.477...
4	0.602...
5	0.698...
6	0.778...
7	0.845...
8	0.903...
9	0.954...
10	1

x	$g(x) = 2 \log_{10} x$
−1	undefined
0	undefined
1	0
2	0.602...
3	0.954...
4	1.204...
5	1.397...
6	1.556...
7	1.690...
8	1.806...
9	1.908...
10	2

x	$h(x) = 5 \log_{10} x$
−1	undefined
0	undefined
1	0
2	1.505...
3	2.385...
4	3.010...
5	3.494...
6	3.890...
7	4.225...
8	4.515...
9	4.771...
10	5







C. Examine the graph of each function, and state the following characteristics:

- the number of x-intercepts → one
- the y-intercept → none (vertical asymptote: $x = 0$)
- the end behaviour → QIV → QI
- the domain $\{x \in \mathbb{R} \mid x > 0\}$
- the range $y \in \mathbb{R}$

G. On a new screen, graph the function $y = \ln x$ and two other functions of the form $y = a \ln x$, where $a > 0$. Examine the graph of each function, and state the following characteristics:

- the number of x-intercepts → one
- the y-intercept → none
- the end behaviour → QIV → QI
- the domain → $\{x \in \mathbb{R} \mid x > 0\}$
- the range → $y \in \mathbb{R}$

* log functions are just the inverse of exponentials

→ we often use dependent data on the X-axis

$\log = \log_{10}$ common logarithm

$\ln = \log_e$ natural logarithm

A logarithmic function is a function of the form $y = a \log_b x$ where $b > 0$, $b \neq 1$ and $a \neq 0$, and a & b are real numbers. $x > 0$

EXAMPLE 2 Connecting the characteristics of a decreasing natural logarithmic function to its equation and graph

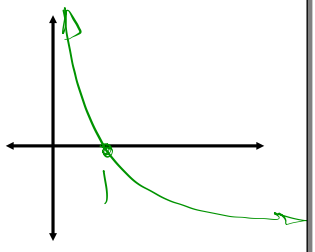
Predict the x-intercept, the number of y-intercepts, the end behaviour, the domain, and the range of the following function:

$$y = -4 \ln x$$

Use the equation of the function to make your predictions. Verify your predictions using graphing technology.

x-ints = one
y-ints = none
E.B. ⇒ QI → QIV

Range: $y \in \mathbb{R}$
domain: $\{x \in \mathbb{R} \mid x > 0\}$

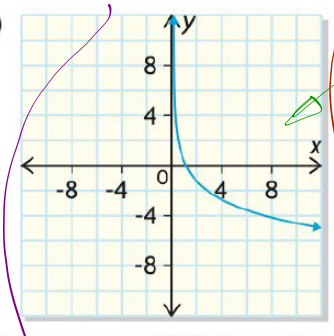


EXAMPLE 3 Matching equations of exponential and logarithmic functions with their graphs

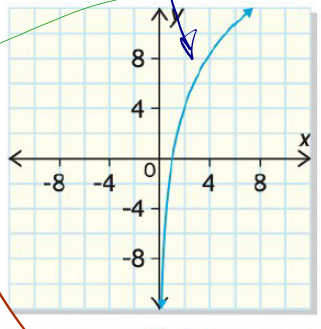
Which function matches each graph below? Provide your reasoning.

i) $y = 5(2)^x$ **ii)** $y = 2(0.1)^x$ **iii)** $y = 6 \log x$ **iv)** $y = -2 \ln x$

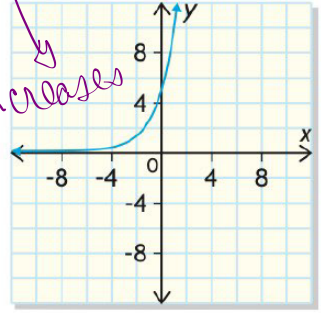
a)



c)

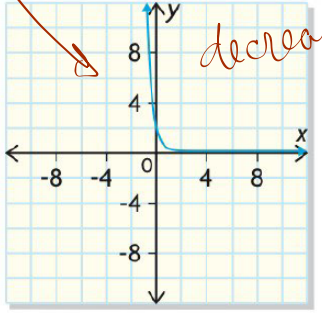


b)



increases

d)



decreases

Read pg. 481 "In Summary"
Homefun: Pg. 461 # 4-6, 8, 12, 17

In Summary

Key Ideas

- A logarithmic function has the form $f(x) = a \log_b x$, where $b > 0$, $b \neq 1$, and $a \neq 0$, and a and b are real numbers.
- All logarithmic functions of the form $f(x) = a \log x$ and $f(x) = a \ln x$ have these characteristics:

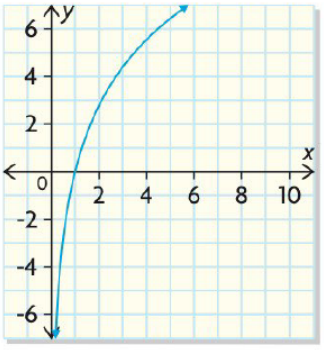
x-Intercept	1
Number of y-Intercepts	0
End Behaviour	The curve extends from quadrant IV to quadrant I or quadrant I to quadrant IV.
Domain	$\{x \mid x > 0, x \in \mathrm{R}\}$
Range	$\{y \mid y \in \mathrm{R}\}$

- All logarithmic functions of the form $f(x) = a \log x$ and $f(x) = a \ln x$ have these unique characteristics:
  - If $a > 0$, the function increases.
  - If $a < 0$, the function decreases.

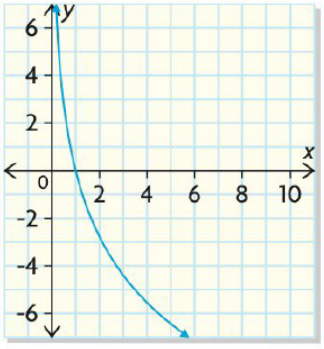
Need to Know

- The graph of a logarithmic function of the form $f(x) = a \log x$ or $f(x) = a \ln x$ will look like one of the following cases:

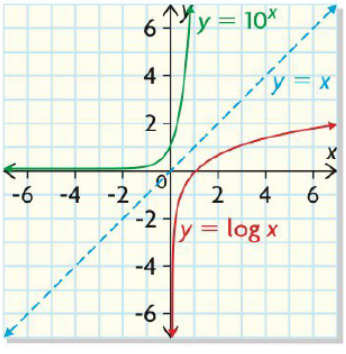
Case 1: an increasing function, where $a > 0$



Case 2: a decreasing function, where $a < 0$



- The graph of $y = \log x$ is a reflection of the graph of $y = 10^x$ about the line $y = x$.



- The graph of $y = \ln x$ is a reflection of the graph of $y = e^x$ about the line $y = x$.