TRICKLE DOWN...

Turn the top word into the bottom word.

You may only change one letter at a time... the results must still be actual words (ex. not voat)

You must try to do it in the least amount of steps.

COAT	PLUG	SLANG
↓	↓	↓
VASE	STAY	TWINE
COST	SLUG	SLING
CAST	SLAG	SWING
CASE	STAG	SWINE
VASE	STAY	TWINE

7.4 Proving Trigonometric Identities

General Steps:

1. Work with one side at a time. Start with the more complicated side.

2. SIMPLIFY... turn everything into sine and cosine (if appropriate)

4. Use trig identities that you know... *pythagorean*, double angle, sum/difference

4. Do the arithmetic if necessary:
   - Add fractions (use a common denominator)
   - Expand using FOIL $(x+a)(x+b)$
   - Factor (don't forget differences of squares)

5. Multiply by a conjugate if you see something like:

$$1 \pm \sin\theta,\ 1 \pm \cos\theta,\ \sin\theta \pm 1 \ \ or \ \ \cos\theta \pm 1$$

if $2+\sqrt{3}$ conjugate is $(2-\sqrt{3})$

if $\cos\theta + 1$ ↳ $\cos\theta - 1$

Example: Prove $\dfrac{\sin 2x}{1+\cos 2x} = \tan x$

$$LS = \frac{2\sin x \cos x}{1+(\cos^2 x - \sin^2 x)}$$

$$= \frac{2\sin x \cos x}{\cos^2 x + (1-\sin^2 x)}$$

$$= \frac{2\sin x \cos x}{\cos^2 x + \cos^2 x}$$

$$= \frac{\cancel{2}\sin x \cancel{\cos x}}{\cancel{2}\cos^{\cancel{2}} x}$$

$$= RS$$

QED

$$\boxed{RS = \frac{\sin x}{\cos x}}$$

$$\boxed{\cos^2 x + \sin^2 x = 1}$$

$$\boxed{\cos^2 x = 1 - \sin^2 x}$$

Example 2: Prove $(\csc x + \cot x)(1 - \cos x) = \sin x$

$$LS = \left(\frac{1}{\sin x} + \frac{\cos x}{\sin x}\right)(1 - \cos x)$$

$$= \left(\frac{1 + \cos x}{\sin x}\right)(1 - \cos x)$$

numerator is a diff. of squares

$$= \frac{1 - \cos^2 x}{\sin x}$$

pythagoras DOS

$$= \frac{\sin^{\cancel{2}} x}{\cancel{\sin x}} = RS$$

$\boxed{QED}$

Example 3: Prove $\cos\left(\frac{\pi}{2} + x\right) = -\sin x$

$$LS = \cos A \cos B - \sin A \sin B$$

$$= \cancel{\cos \frac{\pi}{2}}^{0} \cos x - \cancel{\sin \frac{\pi}{2}}^{1} \sin x$$

$$= 0 - \sin x$$

$$= RS.$$

$\boxed{QED}$

Example 4: Prove

$$\cos(x+y)\cos(x-y) = \cos^2 x\cos^2 y - \sin^2 x\sin^2 y$$

$LS = [\cos x \cos y - \sin x \sin y][\cos x \cos y + \sin x \sin y]$

expand ... ugh!

$= \cos^2 x \cos^2 y +$ ~~need~~ cancels $- \sin^2 x \sin^2 y$

$= RS.$

Example 5: Prove

$$\frac{\cos(x-y)}{\cos(x+y)} = \frac{1+\tan x\tan y}{1-\tan x\tan y}$$

$LS = \dfrac{\cos x \cos y + \sin x \sin y}{\cos x \cos y - \sin x \sin y}$

$RS = \dfrac{1 + \frac{\sin x}{\cos x}\cdot\frac{\sin y}{\cos y}}{1 - \frac{\sin x}{\cos x}\cdot\frac{\sin y}{\cos y}}$

Need C.D.

$= \dfrac{\frac{\cos x \cos y}{\cos x \cos y} + \frac{\sin x \sin y}{\cos x \cos y}}{\frac{\cos x \cos y}{\cos x \cos y} - \frac{\sin x \sin y}{\cos x \cos y}}$

$= \dfrac{\cos x \cos y + \sin x \sin y}{\cancel{\cos x \cos y}} \cdot \dfrac{\cancel{\cos x \cos y}}{\cos x \cos y - \sin x \sin y}$

$= RS$

QED

all $2x$

Example 6: Prove

$$\tan 2x - 2\tan 2x \sin^2 x = \sin 2x$$

$LS = \tan 2x\,(1 - 2\sin^2 x)$

$RS = 2\sin x\cos x$

$= \tan 2x\,(\cos 2x)$

$= \dfrac{\sin 2x}{\cancel{\cos 2x}}\,(\cancel{\cos 2x})$

$= RS$

ex// $\dfrac{1-\cos x}{\sin x} = \dfrac{\sin x}{1+\cos x}$

$LS = \dfrac{1-\cos x}{\sin x} \cdot \dfrac{1+\cos x}{1+\cos x}$

$= \dfrac{1-\cos^2 x}{\sin x\,(1+\cos x)}$ ← Pythag.

$= \dfrac{\sin^{\cancel{2}} x}{\cancel{\sin x}\,(1+\cos x)}$

$= RS$ QED

Homefun:

page 417 #3, 5ace (see example 2 page 413)
9ace, 10ace, 11 first column