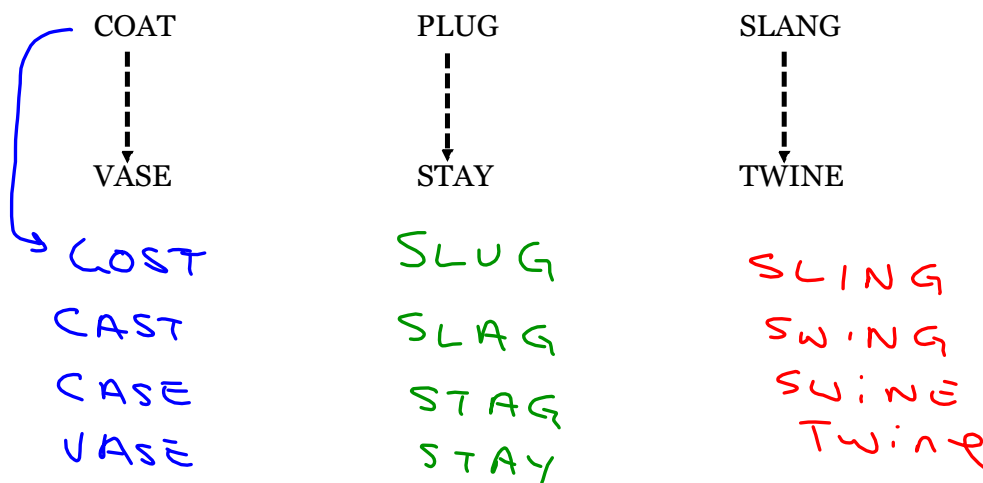


TRICKLE DOWN...

Turn the top word into the bottom word.

You may only change one letter at a time... the results must still be actual words (ex. not voat)

You must try to do it in the least amount of steps.



7.4 Proving Trigonometric Identities

General Steps:

1. Work with one side at a time. Start with the more complicated side.
2. SIMPLIFY... turn everything into sine and cosine (if appropriate)
4. Use trig identities that you know... pythagorean, double angle, sum/difference
4. Do the arithmetic if necessary:
 - Add fractions (use a common denominator)
 - Expand using FOIL $(x+a)(x+b)$
 - Factor (don't forget differences of squares)
5. Multiply by a conjugate if you see something like:

$$1 \pm \sin \theta, 1 \pm \cos \theta, \sin \theta \pm 1 \text{ or } \cos \theta \pm 1$$

Example: Prove $\frac{\sin 2x}{1 + \cos 2x} = \tan x$

if $2 + \sqrt{3}$
 conjugate is
 $(2 - \sqrt{3})$
 if $\cos \theta + 1$
 $\rightarrow \cos \theta - 1$

$$LS = \frac{2 \sin x \cos x}{1 + (\cos^2 x - \sin^2 x)}$$

$$RS = \frac{\sin x}{\cos x}$$

$$= \frac{2 \sin x \cos x}{\cos^2 x + (1 - \sin^2 x)}$$

$$= \frac{2 \sin x \cos x}{\cos^2 x + \cos^2 x}$$

$$= \frac{\cancel{2} \sin x \cancel{\cos x}}{\cancel{2} \cos^{\cancel{2}} x}$$

$$= RS$$

Q.E.D.


$$\cos^2 x + \sin^2 x = 1$$

$$\cos^2 x = 1 - \sin^2 x$$

Example 2: Prove $(\csc x + \cot x)(1 - \cos x) = \sin x$

$$LS = \left(\frac{1}{\sin x} + \frac{\cos x}{\sin x} \right) (1 - \cos x)$$

$$= \left(\frac{1 + \cos x}{\sin x} \right) (1 - \cos x)$$

$$= \frac{1 - \cos^2 x}{\sin x}$$

$$= \frac{\sin^2 x}{\sin x}$$

$$= RS$$

Q.E.D.

numerator
is a diff.
of squares
DOS

Pythagoras

Example 3: Prove $\cos\left(\frac{\pi}{2} + x\right) = -\sin x$

$$LS = \cos A \cos B - \sin A \sin B$$

$$= \cos \frac{\pi}{2} \cos x - \sin \frac{\pi}{2} \sin x$$

$$= 0 - \sin x$$

$$= RS.$$

Q.E.D.

Example 4: Prove

$$\cos(x+y)\cos(x-y) = \cos^2 x \cos^2 y - \sin^2 x \sin^2 y$$

$$L.S. = [\cos x \cos y - \sin x \sin y] [\cos x \cos y + \sin x \sin y]$$

expand ... ugh!

$$= \cos^2 x \cos^2 y + \cancel{\text{red cancels}} - \sin^2 x \sin^2 y$$

$$= R.S.$$

Example 5: Prove

$$\frac{\cos(x-y)}{\cos(x+y)} = \frac{1 + \tan x \tan y}{1 - \tan x \tan y}$$

$$L.S. = \frac{\cos x \cos y + \sin x \sin y}{\cos x \cos y - \sin x \sin y}$$

$$R.S. = \frac{1 + \frac{\sin x}{\cos x} \cdot \frac{\sin y}{\cos y}}{1 - \frac{\sin x}{\cos x} \cdot \frac{\sin y}{\cos y}}$$

Need C.D.

$$= \frac{\frac{\cos x \cos y + \sin x \sin y}{\cos x \cos y}}{\frac{\cos x \cos y - \sin x \sin y}{\cos x \cos y}}$$

$$= \frac{\cos x \cos y + \sin x \sin y}{\cos x \cos y - \sin x \sin y}$$

$$= \frac{\cos x \cos y + \sin x \sin y}{\cancel{\cos x \cos y}} \cdot \frac{\cancel{\cos x \cos y}}{\cos x \cos y - \sin x \sin y}$$

$$= R.S.$$

Q.E.D.

Example 6: Prove

$$\tan 2x - 2 \tan x \sin^2 x = \sin 2x$$

all $2x$

$$LS = \tan 2x (1 - 2 \sin^2 x) \quad RS = 2 \sin x \cos x$$

$$= \tan 2x (\cos 2x)$$

$$= \frac{\sin 2x}{\cos 2x} (\cos 2x)$$

$$= RS$$

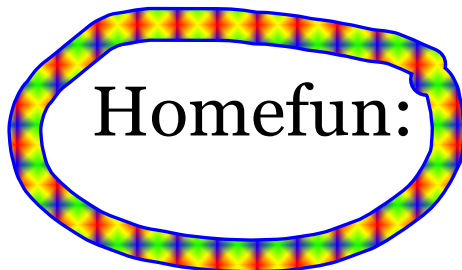
$$\text{ex} // \frac{1 - \cos x}{\sin x} = \frac{\sin x}{1 + \cos x}$$

$$LS = \frac{1 - \cos x}{\sin x} \cdot \frac{1 + \cos x}{1 + \cos x}$$

$$= \frac{1 - \cos^2 x}{\sin x (1 + \cos x)} \quad \leftarrow \text{Pythag.}$$

$$= \frac{\sin^2 x}{\sin x (1 + \cos x)}$$

$$= RS \quad \text{Q.E.D.}$$



Homefun:



page 417



#3, 5ace (see example 2 page 413)

9ace, 10ace, 11 first column

