April 25, 2018

7.4 Solving Systems by Substitution

* Important: This method is most useful when the coefficient on any of the variables in the system equals one.

ex. Solve the following system algebraically:

$$\begin{cases} 2x + y = 3 \quad (1) \\ 4x + 3y = 5 \quad (2) \end{cases}$$

$-2x$

Step 1: isolate the variable with coefficient = 1

(1) $\boxed{y = 3 - 2x}$

Step 2: substitute y for 3−2x in (2)

$4x + 3(3-2x) = 5$

... now solve for x

$4x + 9 - 6x = 5$ $\quad -9$

$\frac{-2x}{-2} = \frac{-4}{-2}$

$\boxed{x = 2}$

Step 3: substitute x = 2 back into either equ^n to get y

(1) $2x + y = 3$

$2(2) + y = 3$

$4 + y = 3$ $\quad -4$

$\boxed{y = -1}$

$\therefore (2, -1)$ is the sol^n

* It is never a bad idea to verify your solution in ***both equations***.

Test the solution in equation (1) & (2) : $(2, -1)$

LS = Left side
RS = Right side

(1) $LS = 2x + y$
$= 2(2) + (-1)$
$= 3$
$= RS$ ✓

(2) $LS = 4x + 3y$
$= 4(2) + 3(-1)$
$= 8 - 3$
$= 5 = RS$ ✓

$\therefore (2,-1)$ is on line (1)

$\therefore (2,-1)$ is also on line (2)

$\therefore (2,-1)$ MUST be the sol^n (pt. of intersection)

* Note: If none of the coefficients equals one this method still works, but it is not always the best option.

ex. Solve the following system algebraically:

$$\begin{cases} 2x + 3y = 12 & (1) \\ 3x - 2y = 3 & (2) \end{cases}$$

$-2x$

Step 1: Try to make a clever decision as to which variable to isolate. Sometimes it is impossible to avoid fractions.

(1) isolate y (there is no great choice)

$$\frac{3y}{3} = \frac{12}{3} - \frac{2x}{3}$$

$$\boxed{y = 4 - \frac{2}{3}x}$$

Step 2: Sub $y = \boxed{4 - \frac{2}{3}x}$ into (2)

$$3x - 2y = 3$$

$$3x - 2\left(4 - \frac{2}{3}x\right) = 3$$

$$\frac{3x}{1}\frac{\times 3}{\times 3} - 8 + \frac{4}{3}x = 3 \quad +8$$

$$\frac{9}{3}x + \frac{4}{3}x = 3 + 8$$

$$\left(\frac{3}{13}\right)\frac{13}{3}x = 11\left(\frac{3}{13}\right)$$

$$\boxed{x = \frac{33}{13}}$$

Step 3: Sub $x = \frac{33}{13}$ into (2)

$$3\left(\frac{33}{13}\right) - 2y = 3$$

$$\frac{99}{13} - 2y = \frac{3 \times 13}{1 \times 13} - \frac{99}{13}$$

$$-2y = \frac{39}{13} - \frac{99}{13}$$

$$\frac{-2y}{-2} = \frac{-60}{13} \div (-2)$$

$$\boxed{y = \frac{30}{13}}$$

ex. Solve the following system algebraically:

$$\begin{cases} \frac{1}{2}x + \frac{2}{3}y = 12 & (1) \\ y = \frac{1}{4}x - \frac{5}{3} & (2) \end{cases}$$

Step 1:

done!

(2) $y = \boxed{\frac{1}{4}x - \frac{5}{3}}$

or multiply each eqn by a number that eliminates the fractions

Step 2: Sub (2) into (1)

$$\frac{1}{2}x + \frac{2}{3}\left(\frac{1}{4}x - \frac{5}{3}\right) = 12$$

$$\frac{1}{2}x + \frac{2}{12}x - \frac{10}{9} = 12$$

$$\left[\frac{1}{2}x + \frac{1}{6}x - \frac{10}{9} = 12\right]18$$

$$9x + 3x - 20 = 216 \quad +20$$

$$\frac{12x}{12} = \frac{236}{12}$$

$$\boxed{x = \frac{59}{3}}$$

Step 3:

Sub $x = \frac{59}{3}$ into (2)

$$y = \frac{1}{4}\left(\frac{59}{3}\right) - \frac{5}{3}\frac{\times 4}{\times 4}$$

$$y = \frac{59}{12} - \frac{20}{12}$$

$$y = \frac{39 \div 3}{12 \div 3}$$

$$\boxed{y = \frac{13}{4}}$$

$\therefore$ Sol^n $\left(\frac{59}{3}, \frac{13}{4}\right)$

Homefun: Pg. 425 # 4ac, 5ac, 7a, 8-12, 15, 19ac, 21

QUIZ Monday 7.1-7.4